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WEAK CONVERGENCE TO A TWO-PARAMETER VOLTERRA-TYPE MULTIFRACTIONAL GAUSSIAN PROCESS IN ANISOTROPIC BESOV SPACES

BY

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Abstract. Under a Besov norm, we give an approximation in law of a two-parameter Volterra-type multifractional Gaussian process. This approximation is constructed from a two-parameter Poisson process.

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1. INTRODUCTION

The fractional Brownian motion (for short, fBm) was first introduced in a Hilbert framework by Kolmogorov [13]. Later, this process was used by Mandelbrot and Van Ness [15] in modeling natural phenomena such as hydrology. The fBm is defined as a continuous real-valued, centered, Gaussian process $\{X_t^\alpha : t \in \mathbb{R}_+\}$ with covariance function

$$\mathbb{E}(X_t^\alpha X_s^\alpha) = \tfrac{1}{2}(|t|^{2\alpha} + |s|^{2\alpha} - |t-s|^{2\alpha}),$$

where the constant $\alpha \in \,]0,1[$ is called the *Hurst parameter*. One of the two-parameter analogues of the fBm known as the fractional Brownian sheet (for short, fBs) with Hurst parameter $(\alpha,\beta) \in (0,1)^2$ is a continuous, real-valued and centered Gaussian process $\{X_{s,t}^{\alpha,\beta} : (s,t) \in [0,1]^2\}$ with covariance function

$$\mathbb{E}(X_{s,t}^{\alpha,\beta} X_{s',t'}^{\alpha,\beta}) = \tfrac{1}{4}(|s'|^{2\alpha} + |s|^{2\alpha} - |s'-s|^{2\alpha})(|t'|^{2\beta} + |t|^{2\beta} - |t'-t|^{2\beta}).$$

Several integral representations of the fBm and the fBs in terms of Wiener integrals have been established. One can divide these representations into three classes: moving average (see Léger and Pontier [14] and Mandelbrot and Van Ness [15]), harmonizable (see Benassi et al. [5] and Ayaché and Léger [1]) and Volterra-type (see Decreusefond and Üstünel [10], Barton and Poor [4] and Feyel and

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de la Pradelle [11]). In this paper, we deal with the Volterra-type representation. For the fBm, that is a stochastic process X^α defined by

$$X_t^\alpha = \int_0^t K_\alpha(t,u)\,dW_u, \quad t \in \mathbb{R}_+, \tag{1.1}$$

where W denotes a standard Brownian motion and K_α is the kernel given by

$$K_\alpha(t,u) = \frac{1}{\Gamma(\alpha-1/2)} u^{1/2-\alpha} \int_u^t x^{\alpha-1/2}(x-u)^{\alpha-3/2}\,dx, \quad 0 \leqslant u \leqslant t,$$

if $\alpha \in (1/2,1)$,

and

$$K_\alpha(t,u) = \frac{2^{-2\alpha}\sqrt{\pi}}{\Gamma(\alpha)\sin\pi\alpha} u^{\alpha-1/2} + \frac{\Gamma(2-\alpha)}{2\Gamma(1-2\alpha)\Gamma(\alpha+1/2)} \times (t-u)^{\alpha-1/2} \int_0^1 (1-x)^{-2\alpha}\left(1-\frac{u}{t}x\right)^{\alpha-1/2} dx, \quad 0 \leqslant u \leqslant t,$$

if $\alpha \in (0,1/2)$.

For the fBs, the Volterra-type representation gives a stochastic process $X^{\alpha,\beta}$ defined by

$$X_{s,t}^{\alpha,\beta} = \int_0^t \int_0^s K_\alpha(t,v) K_\beta(s,u)\,dW_{u,v}, \quad (s,t) \in \mathbb{R}_+^2, \tag{1.2}$$

where $W = \{W_{u,v} : (u,v) \in \mathbb{R}_+^2\}$ is the standard Brownian sheet.

The fact that the Hurst parameter of the fBm is constant restricts its applications in several situations. In order to remedy this, the multifractional Brownian motion (mBm, for short) was introduced by letting the Hurst parameter be a Hölder function with values in $(0,1)$. Depending on the integral representation used for the fBm, different multifractional Gaussian processes were introduced. For example, Peletier and Lévy Vehel [12] and Benassi et al. [5] considered, respectively, the moving average and the harmonizable representation of the fBm. Unlike the case of the fBm, the processes obtained are not equivalent in law [18]. It should also be noted that [7] introduced a Volterra-type multifractional Gaussian process by replacing in the Volterra-type representation of the fBm the Hurst parameter α by a function $t \mapsto \alpha(t)$. More precisely, we take a function parameter $\alpha : [0,\infty[\to (0,1)$ and put

$$X_t^{\alpha(t)} = \int_0^t K_{\alpha(t)}(t,u)\,dW_u, \quad t \in \mathbb{R}_+. \tag{1.3}$$

In this paper, we introduce a two-parameter Volterra-type multifractional Gaussian process which is a centered and continuous process obtained by replacing the parameters α and β in (1.2) by two functions.

More precisely, let $\alpha : [0, \infty[\to]0, 1[$ and $\beta : [0, \infty[\to]0, 1[$ be two functions. The two-parameter Volterra-type multifractional Gaussian process with functional parameters $\alpha(\cdot), \beta(\cdot)$ is the centered Gaussian process $\{X_{s,t}^{\alpha(s),\beta(t)} : (s,t) \in \mathbb{R}_+^2\}$ defined by the following Volterra-type representation:

$$X_{s,t}^{\alpha(s),\beta(t)} = \int_0^t \int_0^s K_{\alpha(s)}(s,u) K_{\beta(t)}(t,v)\, dW_{u,v}, \qquad (s,t) \in \mathbb{R}_+^2, . \tag{1.4}$$

In recent years, many new results on approximation of fBs have been established in the space $C([0,1]^2)$ (for example see [2, 3]) and in Besov spaces (for example see [6, 16, 19]). Since $X_{\cdot}^{\alpha(\cdot),\beta(\cdot)}$ is an extension of fBs, a natural question arises: what processes can approximate $X_{\cdot}^{\alpha(\cdot),\beta(\cdot)}$ in law? We study this question in the case $\alpha(s), \beta(t) > 1/2$ because the two-parameter Volterra-type multifractional Gaussian process is then well defined and it is appropriate for long memory applications.

The aim of our paper is to construct a sequence $\{Y^n(s,t) : (s,t) \in [0,1]^2\}_{n \geqslant 1}$ of processes which approximate $X_{\cdot}^{\alpha(\cdot),\beta(\cdot)}$ in law in the topology of anisotropic Besov spaces. The construction of that sequence is based on $X_{\cdot}^{\alpha(\cdot),\beta(\cdot)}$ and Poisson processes in the plane. The method we use is inspired by Tudor [19]. The novelty of our work compared to [19] is that the two-parameter Volterra-type multifractional Gaussian process has time-dependent Hurst parameters α, β and moreover it is not stationary and its regularity is linked to the Hölder regularity of α and β. We establish our results using new estimation techniques.

The rest of the paper is organized as follows. In Section 2, we recall basic notations on modular Besov spaces. A Kolmogorov criterion and a tightness criterion in these spaces are given. We also prove that the two-parameter Volterra-type multifractional Gaussian process lives in these spaces. Section 3 is devoted to to showing weak convergence of the sequence $\{Y^n : n \geqslant 1\}$ to the two-parameter Volterra-type multifractional Gaussian process.

2. ANISOTROPIC BESOV SPACES

For any $a_i \in \mathbb{R}, i = 1, 2$, and $b \in \mathbb{R}$, let $\overline{a} = (a_1, a_2)$ and $\underline{b} = (b, b)$. For any two functions $\omega_1, \omega_2 : \mathbb{R}_+ \to \mathbb{R}_+$, we put $\omega = \omega_1 \otimes \omega_2$ with $\omega(s,t) = \omega_1(s)\omega_2(t)$ for every $(s,t) \in \mathbb{R}_+^2$.

DEFINITION 2.1. A continuous, non-decreasing function $\omega : [0, \infty[\to [0, \infty[$ is a (β, γ)-*modulus* for $\beta, \gamma > 0$ if there exists a constant $K > 0$ such that for any $j_0 \in \{0, 1, \dots\}$, we have

(i) $\omega(0) = 0, \omega(2t) \leqslant K\omega(t), \forall t \geqslant 0$;

(ii) $\sum_{j_0\leqslant j} 2^{j\beta}\omega(2^{-j}) \leqslant K2^{j_0\beta}\omega(2^{-j_0})$;

(iii) $\sum_{0\leqslant j\leqslant j_0}(2^{-j\beta}/\omega(2^{-j})) \leqslant K(2^{j_0\beta}/\omega(2^{-j_0}))$;
when $\omega(2^{-j_0}) = 0$ for $0 \leqslant j \leqslant j_0, \sum_{0\leqslant j\leqslant j_0}(2^{-j\beta}/\omega(2^{-j})) = \infty$;

(iv) $\sum_{j_0\leqslant j}(2^{-j\gamma}/\omega(2^{-j})) \leqslant K(2^{-j_0\gamma}/\omega(2^{-j_0}))$;
when $\omega(2^{-j_0}) = 0$ for $j_0 \leqslant j, \sum_{j_0\leqslant j}(2^{-j\beta}/\omega(2^{-j})) = \infty$;

(v) $\sum_{0\leqslant j\leqslant j_0} 2^{j\gamma}\omega(2^{-j}) \leqslant K2^{j_0\gamma}\omega(2^{-j_0})$.

This definition leads to the following one:

DEFINITION 2.2. A function $\omega : [0,\infty[^2 \to [0,\infty[$ is said to be a $(\overline{\beta},\overline{\gamma})$-*modulus* if $\omega = \omega_1 \otimes \omega_2$ where ω_i is a (β_i,γ_i)-modulus for $i \in \{1,2\}$.

Let $I = [0,1]^2$. For any $p \in [1,\infty[$, $L^p(I)$ stands for the space of Lebesgue p-integrable functions and $\mathcal{C}(I)$ the space of continuous functions on $I = [0,1]^2$. Let $f : I = [0,1]^2 \to \mathbb{R}$ and $h \in \mathbb{R}$. The *progressive difference* of f in direction e_i $(i = 1,2)$ is defined by

$$\Delta_{h,i}f(z) = \begin{cases} f(z+he_i) - f(z) & \text{if } z, z+he_i \in I, \\ 0 & \text{otherwise,} \end{cases}$$

where $e_1 = (1,0)$ and $e_2 = (0,1)$.

For any $\overline{h} = (h_1,h_2) \in \mathbb{R}^2$,

$$\begin{aligned}\Delta_{\overline{h}}f(z) &= \Delta_{h_1,1} \circ \Delta_{h_2,2}f(z) \\ &= f(z_1+h_1, z_2+h_2) - f(z_1+h_1, z_2) - f(z_1, z_2+h_2) + f(z_1,z_2).\end{aligned}$$

For a Borel function $f \in L^p(I)$ if $1 \leqslant p < \infty$ or $f \in C(I)$ if $p = \infty$, the smoothness of f can be measured by its modulus of continuity in $L^p(I)$-norm defined by

$$\begin{cases} \omega_{p,i}(f,t) = \sup\limits_{|h|\leqslant t} \|\Delta_{h,i}f\|_{L^p(I)} & \text{for } i = 1,2 \text{ and } t > 0; \\ \omega_p(f,\overline{t}) = \sup\limits_{|h_1|\leqslant t_1,\, |h_2|\leqslant t_2} \|\Delta_{\overline{h}}f\|_{L^p(I)} & \text{for } \overline{h} = (h_1,h_2) \text{ and } \overline{t} = (t_1,t_2), \\ & t_i > 0 \text{ for } i \in \{1,2\}. \end{cases}$$

Let ω be a $(\underline{1/p},\underline{1})$-*modulus*, i.e. $\omega = \omega_1 \otimes \omega_2$ where ω_1 and ω_2 are two $(1/p,1)$-moduli. Set

$$\|f\|_p^\omega = \|f\|_{L^p(I)} + \sup_{0<t\leqslant 1} \frac{\omega_{p,1}(f,t)}{\omega(1,t)} + \sup_{0<t\leqslant 1} \frac{\omega_{p,2}(f,t)}{\omega(t,1)} + \sup_{0<t_1,t_2\leqslant 1} \frac{\omega_p(f,\overline{t})}{\omega(t_1,t_2)}.$$

DEFINITION 2.3. Let $1 \leqslant p < \infty$ and let ω be a $(\underline{1/p},\underline{1})$-modulus. The *anisotropic Besov space* is given by

$$B_p^\omega(I) = \{f \in L^p(I) : \|f\|_p^\omega < \infty\}.$$

The space $B_p^{\omega}(I)$ endowed with the norm $\|\cdot\|_p^{\omega}$ is a nonseparable Banach space which contains the separable Banach subspace

$$B_p^{\omega,0}(I) = \{f \in B_p^{\omega}(I) : \omega_{p,1}(f,t) = o(\omega(1,t)) \text{ and } \omega_{p,2}(f,t) = o(\omega(t,1)) \\ \text{as } t \to 0, \text{ and } \omega_p(f,\bar{t}) = o(\omega(t_1,t_2)) \text{ as } t_1 \wedge t_2 \to 0\}.$$

REMARK 2.1. (1) Let $1 \leqslant p < \infty$ and let $\omega = \omega_1 \otimes \omega_2$, $\omega^0 = \omega_1^0 \otimes \omega_2^0$ be two $(1/p, 1)$-moduli such that

$$\lim_{t\to 0} \frac{\omega_1^0(t)}{\omega_1(t)} = \lim_{t\to 0} \frac{\omega_2^0(t)}{\omega_2(t)} = 0.$$

Then we have a continuous embedding

$$B_p^{\omega^0}(I) \hookrightarrow B_p^{\omega,0}(I).$$

(2) If $p = \infty$ and $f \in C(I)$, then $B_{\infty}^{\omega}(I)$ is is exactly the space of functions satisfying the classical Hölder condition with respect to the modulus ω.

For $1 \leqslant p < \infty$ and $\omega^0 = \omega_1^0 \otimes \omega_2^0$ a $(\underline{1/p}, \underline{1})$-modulus, we set

$$\mathcal{F}_p^{\omega^0} = \Bigg\{\omega : \omega \text{ is a } (\underline{1/p}, \underline{1})\text{-modulus such that} \\ \sum_{j_1 \geqslant 0} \sum_{j_2 \geqslant 0} \left(\frac{\omega^0(2^{-j_1}, 2^{-j_2})}{\omega(2^{-j_1}, 2^{-j_2})} \right)^p < \infty \Bigg\}.$$

For any $\omega = \omega_1 \otimes \omega_2 \in \mathcal{F}_p^{\omega^0}$, we set

$$\mathcal{J}_p^{\omega^0}(\omega) = \Bigg\{\omega' = \omega_1' \otimes \omega_2' \in \mathcal{F}_p^{\omega^0} : \frac{\omega_1'}{\omega_1} \text{ and } \frac{\omega_2'}{\omega_2} \text{ increase near } 0 \text{ and} \\ \lim_{t\to 0} \frac{\omega_1'(t)}{\omega_1(t)} = \lim_{t\to 0} \frac{\omega_2'(t)}{\omega_2(t)} = 0 \Bigg\}.$$

We make the following two remarks:

REMARK 2.2. Let $1 \leqslant p < \infty$ and $\omega \in \mathcal{F}_p^{\omega_0}$ be such that $\mathcal{J}_p^{\omega_0}(\omega) \neq \emptyset$, and consider $\omega' = \omega_1' \otimes \omega_2' \in J_p^{\omega_0}(\omega)$. We have the continuous injection

$$B_p^{\omega'}(I) \hookrightarrow B_p^{\omega,0}(I).$$

REMARK 2.3. For $1 \leqslant p < \infty$, $1/p < \alpha_1 < 1$ and $1/p < \alpha_2 < 1$, let $\omega_i^0(t) = t^{\alpha_i}\ell_i(t)$, $t \in [0,1]$, be a regularly varying function with index α_i $(i = 1, 2)$

satisfying the assumptions of [8, Lemma 2.6], where ℓ_i is a slowly varying function which is bounded on $[a,1]$ for all $0<a<1$. Let

$$\omega_\lambda(s,t)=\omega^0(s,t)\bigg(\log\Big(\frac{1}{s}\Big)\log\Big(\frac{1}{t}\Big)\bigg)^\lambda,$$

where $\lambda>1/p$ and $\omega^0(s,t)=\omega_1^0(s)\omega_2^0(t)$. Then $\omega_\lambda\in\mathcal{F}_p^{\omega^0}$ and for any $1/p<\lambda<\lambda'$, we have $\omega_{\lambda'}\in\mathcal{J}_p^{\omega^0}$.

Let us recall the following Kolmogorov-type criterion proved in [16].

LEMMA 2.1 (Kolmogorov criterion). *Let $1\leqslant p<\infty$, let $\omega^0=\omega_1^0\otimes\omega_2^0$ be a $(\underline{1/p},\underline{1})$-modulus and $\{X_{s,t}:(s,t)\in[0,1]^2\}$ be a continuous real valued stochastic process satisfying the following assumptions:*

(i) $X_{0,t}=X_{t,0}=X_{0,0}=0$ *for every* $t\in[0,1]$;

(ii) *there exists a constant $C_p>0$ such that*

$$\big[\mathbb{E}|\Delta_{(s,t)}X_{s',t'}|^p\big]^{1/p}\leqslant C_p\omega_1^0(|s'-s|)\omega_2^0(|t'-t|).$$

Then the following hold:

(a) *For any $\omega\in\mathcal{F}_p^{\omega^0}$, there exists a constant $C(p,\omega^0,\omega)>0$ such that*

$$\forall\lambda>0\quad\mathbb{P}\{\|X\|_p^\omega>\lambda\}\leqslant C(p,\omega^0,\omega)\lambda^{-p}.$$

In particular, $\mathbb{P}\{X\in B_p^\omega(I)\}=1$ for all $\omega\in\mathcal{F}_p^{\omega^0}$.

(b) *For any $\omega\in\mathcal{F}_p^{\omega^0}$ such that $\mathcal{J}_p^{\omega^0}(\omega)\neq\emptyset$, we have $P\{X\in B_p^{\omega,0}(I)\}=1$.*

Now, we can show that the two-parameter type multifractional Gaussian process $X_\cdot^{\alpha(\cdot),\beta(\cdot)}$ lives in anisotropic Besov spaces by using Lemma 2.1. For this, we need the following results.

LEMMA 2.2. *For every $\lambda\in[a,b]\subset\,]1/2,1[$ there exists $\Phi\in L^2(]0,1],\mathbb{R}_+)$ such that for every $u\in\,]0,1]$,*

$$\sup_{\lambda\in[a,b],\,t\in]0,1]}\bigg|\frac{\partial}{\partial\lambda}K_\lambda(t,u)\bigg|\leqslant\Phi(u).$$

Proof. Since $K_\lambda(t,u)=\frac{1}{\Gamma(\lambda-1/2)}u^{1/2-\lambda}\int_u^t x^{\lambda-1/2}(x-u)^{\lambda-3/2}\,dx$, we have

$$\begin{aligned}\frac{\partial}{\partial\lambda}K_\lambda(t,u)&=-\frac{\Gamma'(\lambda-1/2)}{\Gamma(\lambda-1/2)^2}u^{1/2-\lambda}\int_u^t x^{\lambda-1/2}(x-u)^{\lambda-3/2}\,dx\\&\quad+\frac{1}{\Gamma(\lambda-1/2)}\Big\{-\log(u)u^{1/2-\lambda}\int_u^t x^{\lambda-1/2}(x-u)^{\lambda-3/2}\,dx\\&\quad+u^{1/2-\lambda}\int_u^t x^{\lambda-1/2}(x-u)^{\lambda-3/2}(\log(x-u)+\log(x))\,dx\Big\}.\end{aligned}$$

Since $t \mapsto \Gamma(t)$ is of class C^2 on $[0,\infty[$, the functions $\lambda \mapsto -\frac{\Gamma'(\lambda-1/2)}{\Gamma(\lambda-1/2)^2}$ and $\lambda \mapsto \frac{1}{\Gamma(\lambda-1/2)}$ are bounded on $[a,b]$. Moreover, there exist constants $C(a)$ and $C(a,b)$ such that for all $\lambda \in [a,b]$ and $u \in]0,1]$,

$$u^{1/2-\lambda} \int_u^t x^{\lambda-1/2}(x-u)^{\lambda-3/2}\,dx \leqslant c(a)u^{1/2-b}$$

and

$$\Big|\Big\{(-\log(u)u^{1/2-\lambda}) \int_u^t x^{\lambda-1/2}(x-u)^{\lambda-3/2}\,dx + u^{1/2-\lambda} \int_u^t x^{\lambda-1/2}(x-u)^{\lambda-3/2}(\log(x-u)+\log(x))\,dx\Big\}\Big| \leqslant C(a,b)(1 \vee |\log(u)|)u^{1/2-b}.$$

Therefore,

$$\left|\frac{\partial}{\partial\lambda}K_\lambda(t,u)\right| \leqslant k(a)u^{1/2-b} + K(a,b)(1 \vee |\log(u)|)u^{1/2-b} = \Phi(u),$$

where $k(a)$ and $K(a,b)$ are positive constants. ∎

PROPOSITION 2.1.

(a) *For every* $\lambda \in [a,b] \subset]1/2,1[$ *and for all* $t,s \in [0,1]$,

$$\int_0^1 [K_\lambda(t,u) - K_\lambda(s,u)]^2\,du = C_\lambda^2|t-s|^{2\lambda} \leqslant C|t-s|^{2\lambda}, \tag{2.1}$$

where

$$C_\lambda^2 = \frac{\beta(2-2\lambda,\lambda-1/2)}{\lambda(2\lambda-1)(\Gamma(\lambda-1/2))^2} \quad \textit{and} \quad C = \sup_{\lambda\in[a,b]} C_\lambda.$$

(b) *For all* $\lambda,\lambda' \in [a,b] \subset]1/2,1[$ *and* $t \in [0,1]$,

$$\int_0^1 [K_{\lambda'}(t,u) - K_\lambda(t,u)]^2\,du \leqslant C|\lambda'-\lambda|^2. \tag{2.2}$$

Proof. (a) For every $\lambda \in [a,b] \subset]1/2,1[$ and all $t,s \in [0,1]$ we have, by Nualart [17, p. 278, (5.8)],

$$C_\lambda^{-2} \int_0^1 [K_\lambda(t,u) - K_\lambda(s,u)]^2\,du = \mathbb{E}|W_t^\lambda - W_s^\lambda|^2 = |t-s|^{2\lambda},$$

where W^λ is the fBm with Hurst parameter λ.

(b) There exists $\xi = \xi(\lambda,\lambda') \in [\min(\lambda,\lambda'),\max(\lambda,\lambda')]$ such that

$$K_{\lambda'}(t,u) - K_\lambda(t,u) = (\lambda-\lambda')\left|\frac{\partial}{\partial\lambda}K_\lambda(t,u)\right|_{\lambda=\xi=\xi(\lambda,\lambda')}$$

Then, thanks to Lemma 2.2 we get, for all t, λ and λ',

$$\int_0^1 [K_{\lambda'}(t,u) - K_\lambda(t,u)]^2 \, du \leqslant C|\lambda' - \lambda|^2 \int_0^1 \Phi^2(u) \, du,$$

which ends the proof. ∎

Now, we are ready to prove that the two-parameter Volterra-type multifractional Gaussian process lives in a modular Besov space.

For technical reasons, we assume in the remainder of this paper that $\alpha : [0,\infty[\to [a,b] \subset]1/2,1[$ is an η-Hölder function and $\beta : [0,\infty[\to [a,b] \subset]1/2,1[$ is a θ-Hölder function with $\sup_{t\geqslant 0} \alpha(t) < \eta < 1$ and $\sup_{t\geqslant 0} \beta(t) < \theta < 1$. The condition that α and β are Hölder functions plays a crucial role in defining local regularity of the two-parameter Volterra-type multifractional Gaussian process and membership in a modular Besov space. Indeed, the Hölder regularity of α and β determines the uniform Hölder regularity of the trajectories of the process $\{X_{s,t}^{\alpha(s),\beta(t)} : (s,t) \in [0,1]^2\}$ and allows us to precisely locate the process in a modular Besov space.

PROPOSITION 2.2. *Let $k = \min(a,\eta)$, $k' = \min(a,\theta)$ and $\omega^0(s,t) = s^k t^{k'}$. Then for $\lambda > 0$ and $p > \max\{1/k, 1/k', 1/\lambda\}$, the two-parameter Volterra-type multifractional Gaussian process $\{X_{s,t}^{\alpha(s),\beta(t)} : (s,t) \in [0,1]^2\}$ defined by* (1.4) *satisfies*

$$\mathbb{P}\{X_\cdot^{\alpha(\cdot),\beta(\cdot)} \in B_p^{\omega_\lambda,0}([0,1]^2)\} = 1,$$

where $\omega_\lambda(s,t) = \omega^0(s,t)(\log(1/s)\log(1/t))^\lambda$.

Proof. We will extend the kernels $K_{\alpha(s)}(s,x)$ and $K_{\beta(t)}(t,y)$ over all $[0,1]$ by putting

$$\tilde{K}_{\alpha(s)}(s,x) = \begin{cases} K_{\alpha(s)}(s,x) & \text{if } x < s, \\ 0 & \text{if } x \geqslant s, \end{cases}$$

and

$$\tilde{K}_{\beta(t)}(t,y) = \begin{cases} K_{\beta(t)}(t,y) & \text{if } y < t, \\ 0 & \text{if } y \geqslant t. \end{cases}$$

For every $]s,s'] \times]t,t'] \subset [0,1]^2$, we denote by

$$\Delta_{(s,t)} X_{s',t'}^{\alpha(s'),\beta(t')} = X_{s',t'}^{\alpha(s'),\beta(t')} - X_{s',t}^{\alpha(s'),\beta(t)} - X_{s,t'}^{\alpha(s),\beta(t')} + X_{s,t}^{\alpha(s),\beta(t)}$$

the rectangular increments of the two-parameter Volterra-type multifractional Gaussian process $X^{\alpha(\cdot),\beta(\cdot)}_{\cdot}$. Put

$$\begin{aligned}
A_1 &= X^{\alpha(s),\beta(t)}_{s',t'} - X^{\alpha(s),\beta(t)}_{s,t'} - X^{\alpha(s),\beta(t)}_{s',t} + X^{\alpha(s),\beta(t)}_{s,t},\\
A_2 &= X^{\alpha(s),\beta(t')}_{s',t'} - X^{\alpha(s),\beta(t')}_{s,t'} - X^{\alpha(s),\beta(t)}_{s',t'} + X^{\alpha(s),\beta(t)}_{s,t'},\\
A_3 &= X^{\alpha(s'),\beta(t')}_{s',t} - X^{\alpha(s'),\beta(t)}_{s',t} - X^{\alpha(s),\beta(t')}_{s',t} + X^{\alpha(s),\beta(t)}_{s',t},\\
A_4 &= X^{\alpha(s'),\beta(t')}_{s',t'} - X^{\alpha(s),\beta(t')}_{s',t'} - X^{\alpha(s'),\beta(t')}_{s',t} + X^{\alpha(s),\beta(t')}_{s',t}.
\end{aligned}$$

Since $\Delta_{(s,t)}X^{\alpha(s'),\beta(t')}_{s',t'} = A_1 + A_2 + A_3 + A_4$, we have

$$\mathbb{E}|\Delta_{(s,t)}X^{\alpha(s'),\beta(t')}_{s',t'}|^2 \leqslant 4(\mathbb{E}|A_1|^2 + \mathbb{E}|A_2|^2 + \mathbb{E}|A_3|^2 + \mathbb{E}|A_4|^2). \tag{2.3}$$

Let us deal with the right side of (2.3). For $(s,t) \in [0,1]^2$ given, the random field $(X^{\alpha(s),\beta(t)}_{u,v})_{(u,v)\in[0,1]^2}$ is a fractional Brownian sheet. Therefore, it is self-similar and has stationary increments. Therefore, by Proposition 2.1,

$$\mathbb{E}|A_1|^2 = C_0|s'-s|^{2\alpha(s)}|t'-t|^{2\beta(t)}.$$

For A_2, we have

$$\begin{aligned}
A_2 &= \int_{[0,1]^2} [\tilde{K}_{\alpha(s)}(s',u)\tilde{K}_{\beta(t')}(t',v) - \tilde{K}_{\alpha(s)}(s,u)\tilde{K}_{\beta(t')}(t',v)\\
&\quad - \tilde{K}_{\alpha(s)}(s',u)\tilde{K}_{\beta(t)}(t',v) + \tilde{K}_{\alpha(s)}(s,u)\tilde{K}_{\beta(t)}(t',v)]\, dW_{u,v}\\
&= \int_{[0,1]^2} [\tilde{K}_{\alpha(s)}(s',u) - \tilde{K}_{\alpha(s)}(s,u)][\tilde{K}_{\beta(t')}(t',v) - \tilde{K}_{\beta(t)}(t',v)]\, dW_{u,v}.
\end{aligned} \tag{2.4}$$

Since A_2 is a centered Gaussian random variable, by the isometry property and Proposition 2.1 there exists a constant $C_1 > 0$ such that

$$\begin{aligned}
&\mathbb{E}(A_2^2)\\
&= \Big(\int_0^1 [\tilde{K}_{\alpha(s)}(s',u) - \tilde{K}_{\alpha(s)}(s,u)]^2\, du\Big) \times \Big(\int_0^1 [\tilde{K}_{\beta(t')}(t',v) - \tilde{K}_{\beta(t)}(t',v)]^2\, dv\Big)\\
&\leqslant C_1|s'-s|^{2\alpha(s)}|\beta(t') - \beta(t)|^2.
\end{aligned}$$

Since β is a θ-Hölder function, there exists a constant $C_1(\theta) > 0$ such that

$$\mathbb{E}(A_2^2) \leqslant C_1(\theta)|s'-s|^{2\alpha(s)}|t'-t|^{2\theta}.$$

In the same way, one can prove that there exist $C_2(\eta,\theta), C_3(\eta) > 0$ such that

$$\begin{aligned}
\mathbb{E}(A_3^2) &\leqslant C_2(\eta,\theta)|s'-s|^{2\eta}|t'-t|^{2\theta},\\
\mathbb{E}(A_4^2) &\leqslant C_3(\eta)|s'-s|^{2\eta}|t'-t|^{2\beta(t')}.
\end{aligned}$$

Therefore, in view of (2.3), we have

$$\mathbb{E}|\Delta_{(s,t)}X^{\alpha(s'),\beta(t')}_{s',t'}|^2 \leqslant C(\eta,\theta)|s'-s|^{2k}|t'-t|^{2k'},$$

where $k=\min(a,\eta)$ and $k'=\min(a,\theta)$.

Since $\{X^{\alpha(s),\beta(t)}_{(s,t)} : (s,t)\in[0,1]^2\}$ is a Gaussian process, $\Delta_{(s,t)}X^{\alpha(s'),\beta(t')}_{s',t'}$ is a Gaussian random variable. Therefore, $\mathbb{E}|\Delta_{(s,t)}X^{\alpha(s'),\beta(t')}_{s',t'}|^p$ is proportional to $(\mathbb{E}|\Delta_{(s,t)}X^{\alpha(s'),\beta(t')}_{s',t'}|^2)^{p/2}$. It follows that there exists a constant $C(p,\theta,\eta)>0$ such that

$$\mathbb{E}|\Delta_{(s,t)}X^{\alpha(s'),\beta(t')}_{s',t'}|^p \leqslant C(p,\theta,\eta)|s'-s|^{pk}|t'-t|^{pk'}.$$

By Lemma 2.1, we deduce that

$$\mathbb{P}\{X^{\alpha(\cdot),\beta(\cdot)}_{\cdot}\in B_p^{\omega_\lambda,0}([0,1]^2)\}=1$$

with $\omega_\lambda(s,t)=\omega_{k,k'}(s,t)(\log(1/s)\log(1/t))^\lambda$ where $\omega_{k,k'}(s,t)=|s|^k|t|^{k'}$. ∎

To end this section, let us recall the following two lemmas proved in [16].

LEMMA 2.3. *Let $1\leqslant p<\infty$ and $\omega=\omega_1\otimes\omega_2$, $\omega'=\omega_1'\otimes\omega_2'$ be two $(1/p,1)$-moduli such that ω_1'/ω_1 and ω_2'/ω_2 are increasing functions near 0 and*

$$\lim_{t\to0}\frac{\omega_1'(t)}{\omega_1(t)}=\lim_{t\to0}\frac{\omega_2'(t)}{\omega_2(t)}=0. \tag{4.4}$$

Then the space $B_p^{\omega'}(I)$ is compactly embedded in $B_p^{\omega,0}(I)$.

REMARK 2.4. Let $1\leqslant p<\infty$ and $\omega\in\mathcal{F}_p^{\omega_0}$ be such that $\mathcal{J}_p^{\omega_0}(\omega)\neq\emptyset$ and consider $\omega'=\omega_1'\otimes\omega_2'\in J_p^{\omega_0}(\omega)$. Then the space $B_p^{\omega'}(I)$ is compactly embedded in $B_p^{\omega,0}(I)$.

LEMMA 2.4 (Tightness criterion). *Let $1\leqslant p<\infty$, let $\omega^0=\omega_1^0\otimes\omega_2^0$ be a $(\underline{1/p},\underline{1})$-modulus and let $(\{X^n_{s,t} : (s,t)\in[0,1]^2\})_{n\geqslant1}$ be a sequence of real continuous stochastic processes satisfying*

(i) $X^n_{0,t}=X^n_{t,0}=X^n_{0,0}=0$;

(ii) *there exists a constant $C_p>0$ such that for any $[s,s']\times[t,t']\in[0,1]^2$,*

$$[\mathbb{E}|\Delta_{(s,t)}X^n_{s',t'}|^p]^{1/p}\leqslant C_p\omega_1^0(|s'-s|)\omega_2^0(|t'-t|).$$

Then the sequence $(\{X^n_{s,t} : (s,t)\in[0,1]^2\})_{n\geqslant1}$ of stochastic processes is tight in $B_p^{\omega,0}(I)$ for any $\omega=\omega_1\otimes\omega_2\in\mathcal{F}_p^{\omega^0}$ such that $\mathcal{J}_p^{\omega^0}(\omega)\neq\emptyset$.

3. CONVERGENCE IN LAW TO THE TWO-PARAMETER VOLTERRA-TYPE MULTIFRACTIONAL GAUSSIAN PROCESS

In this section we consider the two-parameter Volterra-type multifractional Gaussian process defined by

$$X_{s,t}^{\alpha(s),\beta(t)} = \int_0^t \int_0^s K_{\alpha(s)}(s,v) K_{\beta(t)}(t,u)\, dW_{u,v} \tag{3.1}$$

with $\alpha, \beta : \mathbb{R} \to [a,b] \subset (1/2,1)$ and

$$K_{\alpha(s)}(s,u) = \frac{u^{1/2-\alpha(s)}}{\Gamma(\alpha(s)-1/2)} \int_u^s x^{\alpha(s)-1/2}(x-u)^{\alpha(s)-3/2}\, dx, \quad 0 \leqslant u \leqslant s,$$
$$K_{\beta(t)}(t,v) = \frac{v^{1/2-\beta(t)}}{\Gamma(\beta(t)-1/2)} \int_v^t x^{\beta(t)-1/2}(x-v)^{\beta(t)-3/2}\, dx, \quad 0 \leqslant v \leqslant t.$$

To construct an approximation to $X_{\cdot}^{\alpha(\cdot),\beta(\cdot)}$, we note that Tudor [19] proved that the continuous process $\{G^n\}_{n\geqslant 1}$ given by

$$G_{s,t}^n = \int_0^t \int_0^s K_\alpha(s,x) K_\beta(t,y) \theta_n(x,y)\, dx\, dy, \quad \forall (s,t) \in [0,1]^2, \tag{3.2}$$

under certain conditions, converges weakly in an anisotropic Besov space to the fBs $\{B_{s,t}^{\alpha,\beta} = \int_0^t \int_0^s K_\alpha(s,x) K_\beta(t,y)\, dW_{x,y} : (s,t) \in [0,1]^2\}$ as $n \to \infty$, where W is a standard Brownian sheet,

$$\theta_n(x,y) = n\sqrt{xy}(-1)^{N(x\sqrt{n},\, y\sqrt{n})},$$

and $\{N(x,y) : (x,y) \in \mathbb{R}_+^2\}$ is the standard two-parameter Poisson process. Inspired by this work, based on the representation (3.1), we define a sequence of processes as follows:

$$Y_{s,t}^n = \int_0^t \int_0^s K_{\alpha(s)}(s,x) K_{\beta(t)}(t,y) \theta_n(x,y)\, dx\, dy. \tag{3.3}$$

We have the following main result.

THEOREM 3.1. *Let $k = \min(a,\eta)$ and $k' = \min(a,\theta)$. For any $\lambda > 0$ and $p > \max(1/k, 1/k', 1/\lambda)$, the sequence $\{Y_{s,t}^n : (s,t) \in [0,1]^2\}_{n\geqslant 1}$ of processes given by* (3.3) *converges weakly, in the separable Banach space $B_p^{\omega_\lambda,0}([0,1]^2)$, as $n \to \infty$, to the two-parameter Volterra multifractional Gaussian process $\{X_{s,t}^{\alpha(s),\beta(t)} : (s,t) \in [0,1]^2\}$, where $\omega_\lambda(s,t) = s^k t^{k'} (\log(1/s)\log(1/t))^\lambda$.*

Proof. Let C_p denote a positive constant depending on p and independent of n which may vary from line to line.

We extend the kernels $K_{\alpha(s)}(s,x)$ and $K_{\beta(t)}(t,y)$ onto the whole interval $[0,1]$ by putting

$$\tilde{K}_{\alpha(s)}(s,x) = \begin{cases} K_{\alpha(s)}(s,x) & \text{if } x < s, \\ 0 & \text{if } x \geqslant s, \end{cases}$$

$$\tilde{K}_{\beta(t)}(t,y) = \begin{cases} K_{\beta(t)}(t,y) & \text{if } y < t, \\ 0 & \text{if } y \geqslant t. \end{cases}$$

For any $[s,s'] \times [t,t'] \in [0,1]^2$, put

$$A_{1,n} = \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s',x) - \tilde{K}_{\alpha(s')}(s,x)][\tilde{K}_{\beta(t')}(t',y) - \tilde{K}_{\beta(t')}(t,y)]\theta_n(x,y)\,dx\,dy,$$

$$A_{2,n} = \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s',x) - \tilde{K}_{\alpha(s')}(s,x)][\tilde{K}_{\beta(t')}(t,y) - \tilde{K}_{\beta(t)}(t,y)]\theta_n(x,y)\,dx\,dy,$$

$$A_{3,n} = \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s,x) - \tilde{K}_{\alpha(s)}(s,x)][\tilde{K}_{\beta(t')}(t',y) - \tilde{K}_{\beta(t')}(t,y)]\theta_n(x,y)\,dx\,dy,$$

$$A_{4,n} = \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s,x) - \tilde{K}_{\alpha(s)}(s,x)][\tilde{K}_{\beta(t')}(t,y) - \tilde{K}_{\beta(t)}(t,y)]\theta_n(x,y)\,dx\,dy.$$

We have

$$Y^n_{s,t} = \int\limits_{[0,1]^2} \tilde{K}_{\alpha(s)}(s,x)\tilde{K}_{\beta(t)}(t,y)\theta_n(x,y)\,dx\,dy$$

and

$$\begin{aligned} \Delta_{(s,t)}Y^n_{s',t'} &= Y^n_{s',t'} - Y^n_{s',t} - Y^n_{s,t'} + Y^n_{s,t} \\ &= \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s',x) - \tilde{K}_{\alpha(s)}(s,x)][\tilde{K}_{\beta(t')}(t',y) - \tilde{K}_{\beta(t)}(t,y)]\theta_n(x,y)\,dx\,dy \\ &= A_{1,n} + A_{2,n} + A_{3,n} + A_{4,n}. \end{aligned}$$

Since the Besov norms are increasing in p, it suffices to prove the theorem for even p.

By Lemma 2.4, to establish the tightness of $\{Y^n : n \in \mathbb{N}\}$, it suffices to show that for any even p, there exists a constant $C_p > 0$ such that

$$\mathbb{E}|\Delta_{(s,t)}Y^n_{s',t'}|^p \leqslant C_p(s'-s)^{pk}(t'-t)^{pk'}. \tag{3.4}$$

By [3, Lemma 3.2], it suffices to establish (3.4) for $2s > s' > s > 0$ and $2t > t' > t > 0$.

We have

$$\mathbb{E}|\Delta_{(s,t)}Y^n_{s',t'}|^p \leqslant 4^{p-1}(\mathbb{E}|A_{1,n}|^p + \mathbb{E}|A_{2,n}|^p + \mathbb{E}|A_{3,n}|^p + \mathbb{E}|A_{4,n}|^p). \tag{3.5}$$

Let us estimate the terms of the right member of (3.5). We have

$$\begin{aligned}
&\mathbb{E}|A_{2,n}|^p \\
&= n^p\mathbb{E}\Big[\int\limits_{[0,1]^2}[\tilde{K}_{\alpha(s')}(s',x) - \tilde{K}_{\alpha(s')}(s,x)][\tilde{K}_{\beta(t')}(t,y) - \tilde{K}_{\beta(t)}(t,y)] \\
&\qquad\qquad \times \sqrt{xy}(-1)^{N_n(x,y)}\,dx\,dy\Big]^p \\
&= n^p\mathbb{E}\Big[\int\limits_{[0,1]^{2p}}\prod_{i=1}^{p}\Big([\tilde{K}_{\alpha(s')}(s',x_i) - \tilde{K}_{\alpha(s')}(s,x_i)][\tilde{K}_{\beta(t')}(t,y_i) - \tilde{K}_{\beta(t)}(t,y_i)] \\
&\qquad\qquad \times \sqrt{x_iy_i}(-1)^{N_n(x_i,y_i)}\Big)\,dx_1\ldots dy_p\Big],
\end{aligned}$$

where $N_n(x,y) = N(x\sqrt{n}, y\sqrt{n})$.

First, we have

$$\begin{aligned}
N_n(x_i,y_i) &= \Delta_{(0,0)}N_n(x_i,y_i) \\
&= \Delta_{s,t}N_n(x_i,t_i) + \Delta_{s,0}N_n(x_i,t) + \Delta_{0,t}N_n(s,y_i) + N_n(s,t).
\end{aligned} \tag{3.6}$$

We now obtain a bound for the expectation of the random part of the last expression. By (3.6), we have

$$(-1)^{\sum_{i=1}^p N_n(x_i,y_i)} = (-1)^{\sum_{i=1}^p \Delta_{(0,0)}N_n(x_i,y_i)}$$

and

$$\begin{aligned}
\sum_{i=1}^{p}\Delta_{(0,0)}N_n(x_i,y_i) &= \sum_{i=1}^{p}\Delta_{(s,t)}N_n(x_i,y_i) + \sum_{i=1}^{p}\Delta_{(s,0)}N_n(x_i,t)t) \\
&\quad + \sum_{i=1}^{p}\Delta_{(0,t)}N_n(s,y_i) + pN_n(s,
\end{aligned} \tag{3.7}$$

and since p is even, we have

$$\begin{aligned}
&(-1)^{\sum_{i=1}^p \Delta_{(0,0)}N_n(x_i,y_i)} \\
&\quad = (-1)^{\sum_{i=1}^p \Delta_{(s,t)}N_n(x_i,y_i)}(-1)^{\sum_{i=1}^p \Delta_{(s,0)}N_n(x_i,t)}(-1)^{\sum_{i=1}^p \Delta_{(0,t)}N_n(s,y_i)}.
\end{aligned} \tag{3.8}$$

Since for all x_i, y_i, x_j, y_k, the three intervals $((s,t),(x_i,y_i)]$, $((s,0),(x_j,t)]$ and $((0,t),(s,y_k)]$ are disjoint, by using the independence of increments of Poisson processes, the corresponding increments are independent. Moreover, if we majorize in (3.8) the expectation of the first factor by 1 we will obtain

$$\begin{aligned}
&\big|\mathbb{E}\big[(-1)^{\sum_{i=1}^p \Delta_{(0,0)}N_n(x_i,y_i)}\big]\big| \\
&\qquad \leqslant \big|\mathbb{E}\big[(-1)^{\sum_{i=1}^p \Delta_{s,0}N_n(x_i,t)}\big]\big|\,\big|\mathbb{E}\big[(-1)^{\sum_{i=1}^p \Delta_{0,t}N_n(s,y_i)}\big]\big|.
\end{aligned}$$

We know that when Z is distributed as a Poisson random variable with mean λ ($Z \sim \text{Poiss}(\lambda)$) then $\mathbb{E}[(-1)^Z] = \exp(-2\lambda)$. It follows that

$$\begin{aligned}\big|\mathbb{E}\big[(-1)^{\sum_{i=1}^p \Delta_{(0,0)} N_n(x_i,y_i)}\big]\big| &\leqslant \exp\{-2nt[(x_{(p)} - x_{(p-1)}) + \cdots + (x_{(2)} - x_{(1)})]\} \\ &\quad \times \exp\{-2ns[(y_{(p)} - y_{(p-1)}) + \cdots + (y_{(2)} - y_{(1)})]\},\end{aligned}$$

where $x_{(1)}, \ldots, x_{(p)}$ and $y_{(1)}, \ldots, y_{(p)}$ are the variables $x_1, \ldots, x_p$ and $y_1, \ldots, y_p$ arranged in increasing order.

Using now the fact that $2s > s' > s > 0$ and $2t > t' > t > 0$, we obtain

$$\begin{aligned}\big|\mathbb{E}\big[(-1)^{\sum_{i=1}^p \Delta_{(0,0)} N_n(x_i,y_i)}\big]\big| &\leqslant \exp\{-nt'[(x_{(p)} - x_{(p-1)}) + \cdots + (x_{(2)} - x_{(1)})]\} \\ &\quad \times \exp\{-ns'[(y_{(p)} - y_{(p-1)}) + \cdots + (y_{(2)} - y_{(1)})]\} \\ &\leqslant \exp\{-n[(x_{(p)} - x_{(p-1)})y_{(p-1)} + \cdots + (x_{(2)} - x_{(1)})y_{(1)}]\} \\ &\quad \times \exp\{-n[(y_{(p)} - y_{(p-1)})x_{(p-1)} + \cdots + (y_{(2)} - y_{(1)})x_{(1)}]\}.\end{aligned}$$

Therefore,

$$\begin{aligned}&\mathbb{E}|A_{2,n}|^p \\ &\leqslant (p!)^2 n^p \\ &\times \int_{[0,1]^{2p}} \Big(\prod_{i=1}^p \big(|\tilde{K}_{\alpha(s')}(s', x_i) - \tilde{K}_{\alpha(s')}(s, x_i)|\, |\tilde{K}_{\beta(t')}(t, y_i) - \tilde{K}_{\beta(t)}(t, y_i)| \sqrt{x_i, y_i} \big) \\ &\quad \times \exp\{-n[(x_p - x_{p-1})y_{p-1} + \cdots + (x_2 - x_1)y_1]\} \\ &\quad \times \exp\{-n[(y_p - y_{p-1})x_{p-1} + \cdots + (y_2 - y_1)x_1]\} \chi_{\{x_1 \leqslant \cdots \leqslant x_p\}} \chi_{\{y_1 \leqslant \cdots \leqslant y_p\}} \Big) \\ &\qquad\qquad \times\, dx_1 \ldots dx_p\, dy_1 \ldots dy_p \\ &\leqslant C_p \Big(n^2 \int_{[0,1]^4} |\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)|\, |\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)| \\ &\quad \times |\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)|\, |\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)| \sqrt{x_1 x_2 y_1 y_2} \\ &\quad \times \exp\{-n(x_2 - x_1)y_1 - n(y_2 - y_1)x_1\} \chi_{\{x_1 \leqslant x_2\}} \chi_{\{y_1 \leqslant y_2\}} \}\, dx_1\, dx_2\, dy_1\, dy_2 \Big)^{p/2}.\end{aligned} \tag{3.9}$$

Now, we divide the integration region into two parts:

$$A = \{x_1 \leqslant x_2 \leqslant 2x_1,\ y_1 \leqslant y_2 \leqslant 2y_1\} \quad \text{and} \quad A^c.$$

The integral in of (3.9) over A can be majorized by

$$C_p n^2 \int\limits_{[0,1]^4} |\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)|\, |\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)| \\ \times |\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)|\, |\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)| x_1 y_1 \\ \times \exp\{-2n[(x_2 - x_1)y_1 + (y_2 - y_1)x_1]\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}}\, dx_1\, dx_2\, dy_1\, dy_2.$$

By using the elementary inequality $ab \leqslant \frac{1}{2}(a^2 + b^2)$, we obtain

$$C_p n^2 \int\limits_{[0,1]^4} |\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)|\, |\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)| \\ \times |\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)|\, |\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)| x_1 y_1 \\ \times \exp\{-2n[(x_2 - x_1)y_1 + (y_2 - y_1)x_1]\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}}\, dx_1\, dx_2\, dy_1\, dy_2 \\ \leqslant C_p n^2 \int\limits_{[0;1]^4} [\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)]^2 [\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)]^2 \\ \times x_1 y_1 \exp\{-2n[(x_2 - x_1)y_1 + (y_2 - y_1)x_1]\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}}\, dx_1\, dx_2\, dy_1\, dy_2 \\ + C_p n^2 \int\limits_{[0,1]^4} [\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)]^2 [\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)]^2 x_1 y_1 \\ \times \exp\{-2n[(x_2 - x_1)y_1 + (y_2 - y_1)x_1]\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}}\, dx_1\, dx_2\, dy_1\, dy_2.$$

Let

$$I_1 = C_p n^2 \int\limits_{[0,1]^4} [\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)]^2 [\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)]^2 \\ \times x_1 y_1 \exp\left\{-\frac{n}{2}[(x_2 - x_1)y_1 + (y_2 - y_1)x_1]\right\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}}\, dx_1\, dx_2\, dy_1\, dy_2$$

and

$$I_2 = C_p n^2 \int\limits_{[0,1]^4} [\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)]^2 [\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)]^2 \\ \times x_1 y_1 \exp\left\{-\frac{n}{2}[(x_2 - x_1)y_1 + (y_2 - y_1)x_1]\right\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}}\, dx_1\, dx_2\, dy_1\, dy_2.$$

The terms I_1 and I_2 can be treated in a similar way. In I_1, we integrate with respect to x_2 and then with respect to y_1, and in I_2 we integrate with respect to y_2 and then with respect to x_1. It follows that

$$I_1 \leqslant C_p \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)]^2 [\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)]^2\, dx_1 dy_2.$$

and

$$I_2 \leqslant C_p \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)]^2 [\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)]^2 \, dx_2 \, dy_1.$$

We can conclude that

$$\begin{aligned}
&C_p n^2 \int\limits_{[0,1]^4} |\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)| \, |\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)| \\
&\quad \times |\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)| \, |\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)| x_1 y_1 \\
&\quad \times \exp\{-n[(x_2 - x_1)y_1 + (y_2 - y_1)x_1]\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}} \, dx_1 \, dx_2 \, dy_1 \, dy_2 \\
&\leqslant C_p \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)]^2 [\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)]^2 \, dx_2 \, dy_1.
\end{aligned}$$

Now, we consider the integral in (3.9) over A^c. This region is the union of $B_1 = \{x_1 \leqslant x_2, y_1 \leqslant y_2, x_2 > 2x_1\}$ and $B_2 = \{x_1 \leqslant x_2, y_1 \leqslant y_2, y_2 > 2y_1\}$. We will deal with the integral over B_2; the other one is analogous. On B_2, we have

$$\begin{aligned}
2(y_2 - y_1)x_1 + 2(x_2 - x_1)y_1 &\geqslant (y_2 - y_1)x_1 + x_1 y_1 + 2(x_2 - x_1)y_1 \\
&\geqslant \tfrac{1}{2}(y_2 - y_1)x_1 + x_1 y_1 + \tfrac{1}{2}(x_2 - x_1)y_1 = \tfrac{1}{2}(y_2 x_1 + y_1 x_2).
\end{aligned}$$

Thus, the integral over B_2 can be bounded by

$$\begin{aligned}
&C_p n^2 \int\limits_{[0,1]^4} |\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)| \, |\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)| \\
&\quad \times |\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)| \, |\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)| \\
&\quad \times \sqrt{x_1 x_2 y_1 y_2} \exp\left\{-\frac{n}{2}(y_2 x_1 + x_2 y_1)\right\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}} \, dx_1 \, dx_2 \, dy_1 \, dy_2 \\
&\leqslant C_p n^2 \int\limits_{[0,1]^4} [\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)]^2 [\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)]^2 \\
&\quad \times x_1 y_1 \exp\left\{-\frac{n}{2}(y_2 x_1 + x_2 y_1)\right\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}} \, dx_1 \, dx_2 \, dy_1 \, dy_2 \\
&\quad + C_p n^2 \int\limits_{[0,1]^4} [\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)]^2 [\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)]^2 x_2 y_2 \\
&\quad \times \exp\left\{-\frac{n}{2}(y_2 x_1 + x_2 y_1)\right\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}} \, dx_1 \, dx_2 \, dy_1 \, dy_2 = J_1 + J_2.
\end{aligned} \tag{3.10}$$

In J_1 by integrating with respect to x_2 and then with respect to y_2, and in J_2 by integrating with respect to x_1 and then with respect to y_1, we obtain

$$\begin{aligned}
J_1 &\leqslant C_p \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)]^2 [\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)]^2 \, dx_1 \, dy_1, \\
J_2 &\leqslant C_p \int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)]^2 [\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)]^2 \, dx_1 \, dy_1.
\end{aligned}$$

Therefore, in view of (3.10),

$$\begin{aligned}
C_p n^2 \int\limits_{[0,1]^4} & |\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)|\, |\tilde{K}_{\beta(t')}(t, y_1) - \tilde{K}_{\beta(t)}(t, y_1)| \\
& \times |\tilde{K}_{\alpha(s')}(s', x_2) - \tilde{K}_{\alpha(s')}(s, x_2)|\, |\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)| \\
& \times \sqrt{x_1 x_2 y_1 y_2} \exp\left\{-\frac{n}{4}(y_2 x_1 + x_2 y_1)\right\} \chi_{\{x_1 \leqslant x_2,\, y_1 \leqslant y_2\}}\, dx_1\, dx_2\, dy_1\, dy_2 \\
\leqslant C_p \int\limits_{[0,1]^2} & [\tilde{K}_{\alpha(s')}(s', x_1) - \tilde{K}_{\alpha(s')}(s, x_1)]^2 [\tilde{K}_{\beta(t')}(t, y_2) - \tilde{K}_{\beta(t)}(t, y_2)]^2 \\
& \times\, dx_1\, dy_1.
\end{aligned}$$

Thus, by virtue of (3.9),

$$\begin{aligned}
& \mathbb{E}|A_{2,n}|^p \\
& \leqslant C_p \left(\int\limits_{[0,1]^2} [\tilde{K}_{\alpha(s')}(s', x) - \tilde{K}_{\alpha(s')}(s, x)]^2 [\tilde{K}_{\beta(t')}(t, y) - \tilde{K}_{\beta(t)}(t, y)]^2\, dx\, dy \right)^{p/2}.
\end{aligned}$$

In view of Proposition 2.1, we have

$$\int\limits_{[0,1]} [\tilde{K}_{\alpha(s')}(s', x) - \tilde{K}_{\alpha(s')}(s, x)]^2\, dx \leqslant C|s' - s|^{2\alpha(s')}$$

and

$$\begin{aligned}
\int\limits_{[0,1]} [\tilde{K}_{\beta(t')}(t, y) - \tilde{K}_{\beta(t)}(t, y)]^2 dy & \leqslant C'|\beta(t') - \beta(t)|^2 \\
& \leqslant C'(\theta)|t' - t|^{2\theta},
\end{aligned}$$

which leads to

$$\mathbb{E}|A_{2,n}|^p \leqslant C(p, \theta)|s' - s|^{p\alpha(s')}|t' - t|^{p\theta}.$$

In the same way, one can prove that

$$\mathbb{E}|A_{3,n}|^p \leqslant C(\eta, \theta)|s' - s|^{p\eta}|t' - t|^{p\beta(t')}, \;\; \mathbb{E}|A_{4,n}|^p \leqslant C(\eta)|s' - s|^{p\eta}|t' - t|^{p\theta}$$

and

$$\mathbb{E}|A_{1,n}|^p \leqslant C_p(s' - s)^{\alpha(s)}(t' - t)^{\beta(t)}.$$

Thus, we have

$$\mathbb{E}|\Delta_{s,t} Y^n_{s',t'}|^p \leqslant C(\eta, \theta)|s' - s|^{pk}|t' - t|^{pk'},$$

where $k = \min(a, \eta)$ and $k' = \min(a, \theta)$, so we have proved (3.4).

Now, to conclude the proof, it remains to prove the convergence of finite-dimensional distributions of Y^n to those of $X^{\alpha(\cdot),\beta(\cdot)}$. We must show that $\sum_{j=1}^m a_j Y^n_{s_j,t_j}$ converges in law as $n \to \infty$ to $\sum_{j=1}^m a_j X^{\alpha(s_j),\beta(t_j)}_{s_j,t_j}$ for any real numbers a_j, $j = 1, \dots, m$, and every $(s_j, t_j) \in [0,1]^2$, $j = 1, \dots, m$.

To prove this convergence, we prove the convergence of the associated characteristic functions.

We will use the extension of the kernel $K_{\alpha(s)}(s, \cdot)$ onto $[0,1]$.

Since $1/2 < \alpha(s), \beta(t) < 1$, $K_{\alpha(s)}(s, \cdot)$ and $K_{\beta(t)}(t, \cdot)$ belong to $L^2([0,1])$.

Let us consider for any s_j, t_j some sequences $\{\gamma^{j,\ell}\}_{\ell \geqslant 1}$ and $\{\rho^{j,\ell}\}_{\ell \geqslant 1}$ of elementary functions converging in $L^2([0,1]$, as $\ell \to \infty$, respectively to $K_{\alpha(s_j)}(s_j, \cdot)$ and $K_{\beta(t_j)}(t_j, \cdot)$. Set

$$
\begin{aligned}
\Gamma^{j,l}_n &= \int\limits_{[0,1]^2} \gamma^{j,\ell}(x)\rho^{j,\ell}(y)\theta_n(x,y)\,dx\,dy,\\
\Gamma^{j,l} &= \int\limits_{[0,1]^2} \gamma^{j,\ell}(x)\rho^{j,\ell}(y)\,dW_{x,y},\\
I^\ell_{1,n} &= \big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j Y^n_{s_j,t_j}}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}_n}\big]\big|,\\
I^\ell_{2,n} &= \big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}_n}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}}\big]\big|,\\
I^\ell_{3,n} &= \big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j X^{\alpha(s_j),\beta(t_j)}_{s_j,t_j}}\big]\big|.
\end{aligned}
$$

To show that $\sum_{j=1}^m a_j Y^n_{s_j,t_j}$ converges in law as $n \to \infty$ to $\sum_{j=1}^p a_j X^{\alpha(s_j),\beta(t_j)}_{s_j,t_j}$, for any real a_j, $j = 1, \dots, n$, and any $(s_j, t_j) \in [0,1]^2$, $j = 1, \dots, m$, we must prove that for any $\lambda \in \mathbb{R}$,

$$
\lim_{n\to\infty} \big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j Y^n_{s_j,t_j}}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j X^{\alpha(s_j),\beta(t_j)}_{s_j,t_j}}\big]\big| = 0. \tag{3.11}
$$

Since

$$
\begin{aligned}
&\big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j Y^n_{s_j,t_j}}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j X^{\alpha(s_j),\beta(t_j)}_{s_j,t_j}}\big]\big|\\
&\qquad\leqslant \big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j Y^n_{s_j,t_j}}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}_n}\big]\big|\\
&\qquad\quad + \big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}_n}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}}\big]\big|\\
&\qquad\quad + \big|\mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j \Gamma^{j,\ell}}\big] - \mathbb{E}\big[e^{i\lambda\sum_{j=1}^m a_j X^{\alpha(s_j),\beta(t_j)}_{s_j,t_j}}\big]\big|\\
&\qquad= I^\ell_{1,n} + I^\ell_{2,n} + I^\ell_{3,n},
\end{aligned}
$$

to prove (3.11), it suffices to show that $I^\ell_{1,n}, I^\ell_{2,n}$ and $I^\ell_{3,n}$ converge to 0 as $n \to \infty$ and $\ell \to \infty$.

In view of the mean value theorem, we have

$$I_{1,n}^{\ell} \leqslant C \max_j \mathbb{E}|Y_{s_j,t_j}^n - \Gamma_n^{j,\ell}|.$$

Moreover, for any $j = 1, \dots, m$, we have

$$\begin{aligned}
&\mathbb{E}(Y_{s_j,t_j}^n - \Gamma_n^{j,\ell})^2 \\
&\quad = \mathbb{E}\Big(\int_{[0,1]^2} [K_{\alpha(s_j)}(s_j,x) K_{\beta(t_j)}(t_j,y) - \gamma^{j,\ell}(x)\rho^{j,\ell}(y)]\theta_n(x,y)\,dx\,dy\Big)^2 \\
&\quad \leqslant 2\mathbb{E}\Big(\int_{[0,1]^2} K_{\alpha(s_j)}(s_j,x)[K_{\beta(t_j)}(t_j,y) - \rho^{j,\ell}(y)]\theta_n(x,y)\,dx\,dy\Big)^2 \\
&\qquad + 2\mathbb{E}\Big(\int_{[0,1]^2} [K_{\alpha(s_j)}(s_j,x) - \gamma^{j,\ell}(x)]\rho^{j,\ell}(y)\theta_n(x,y)\,dx\,dy\Big)^2.
\end{aligned}$$

By using [3, Lemma 3.1], the last expression is dominated by

$$\begin{aligned}
&C\Big(\int_{[0,1]^2} (K_{\alpha(s_j)}(s_j,x))^2\,dx\Big)\Big(\int_{[0,1]^2} [K_{\beta(t_j)}(t_j,y) - \rho^{j,\ell}(y)]^2\,dy\Big) \\
&\qquad + C\Big(\int_{[0,1]^2} [K_{\alpha(s_j)}(s_j,x) - \gamma^{j,\ell}(x)]^2\,dx\Big)\Big(\int_{[0,1]^2} (\rho^{j,\ell}(y))^2\,dy\Big),
\end{aligned}$$

which converges to zero as $\ell \to \infty$.

Now, let us deal with $I_{2,n}^{\ell}$. Since $x \mapsto \gamma^{j,\ell}(x)$ and $x \mapsto \rho^{j,\ell}(x)$ are elementary functions, the random variable $\Gamma_n^{j,\ell}$ is a linear combination of the increments of the process

$$y_{s,t}^n = \int_0^t \int_0^s \theta_n(x,y)\,dx\,dy.$$

In view of [2] the sequence $\{y^n : n \in \mathbb{N}\}$ of processes converges in law, as $n \to \infty$, to the Brownian sheet. Thus, $\Gamma_n^{j,\ell}$ converges in law to $\Gamma^{j,\ell}$ as $n \to \infty$.

Therefore, for any $\ell \in \mathbb{N}$ given, $I_{2,n}^{\ell}$ tends to 0 as $n \to \infty$.

Finally, concerning $I_{3,n}^{\ell}$, we have

$$\begin{aligned}
I_{3,n}^{\ell} &\leqslant C \max_j \mathbb{E}|\Gamma^{j,\ell} - X_{s_j,t_j}^{\alpha(s_j),\beta(t_j)}| \\
&= C \max_j \mathbb{E}\Big| \int_{[0,1]^2} [K_{\alpha(s_j)}(s_j,x) K_{\beta(t_j)}(t_j,y) - \gamma^{j,\ell}(x)\rho^{j,\ell}(y)]\,dW_{x,y}\Big| \\
&\leqslant C \max_j \|K_{\alpha(s_j)}(s_j,\cdot) \otimes K_{\beta(t_j)}(t_j,\cdot) - \gamma^{j,\ell} \otimes \rho^{j,\ell}\|_{L^2([0,1]^2)},
\end{aligned}$$

where we have used the properties of the stochastic integral. This last $L^2([0,1]^2)$ norm tends to zero as $\ell \to \infty$. ∎

REFERENCES

[1] A. Ayaché and S. Léger, *Fractional and multifractional Brownian motion*, preprint, 2000.
[2] X. Bardina and M. Jolis, *Weak approximation of the Brownian sheet from the Poisson process in the plane*, Bernoulli 6 (2000), 653–665.
[3] X. Bardina, M. Jolis, and C. A. Tudor, *Weak convergence to the fractional Brownian sheet and other two parameter Gaussian processes*, Statist. Probab. Lett. 65 (2003), 317-329.
[4] R. J. Barton and V. H. Poor, *Signal detection in fractional Gaussian noise*, IEEE Trans. Inform. 34 (1988), 943–959.
[5] A. Benassi, S. Jaffard, and D. Roux, *Elliptic Gaussian random processes*, Rev. Mat. Iberoamer. 13 (1997), 19–96.
[6] B. Bouffoussi et R. Guerbaz, *Convergence vers une classe de processus Gaussiens en norme de Besov anisotropique*, Bull. Sci. Math. 130 (2006), 612–623.
[7] B. Boufoussi, M. Dozzi, and R. D. Marty, *Local time and Tanaka formula for a Volterra-type multifractional Gaussian process*, Bernoulli 16 (2010), 1294–1311.
[8] B. Boufoussi and M. N'zi, *On the regularity of the local times of a class of symmetric Levy processes*, Stoch. Stoch. Rep. 69 (2000), 173–194.
[9] H. Brézis, *Analyse fonctionnelle: Théorie et applications*, Dunod, Paris, 1999.
[10] L. Decreusefond and A. S. Üstünel, *Stochastic analysis of the fractional Brownian motion*, Potential Anal. 10 (1998), 177–214.
[11] D. Feyel and A. de La Pradelle, *On fractional Brownian processes*, Potential Anal. 10 (1999), 273–288.
[12] R. F. Peltier and J. Lévy Véhel, *Multifractional Brownian motion: Definition and preliminary results*, Rapport de recherche de l'INRIA, N. 264, 1995.
[13] A. N. Kolmogorov, *Wienersche Spiralen und einige andere interessante Kurven in Hilbertschen Raum*, C. R. (Doklady) Acad. Sci. USSR (N.S.) 26 (1940), 115–118.
[14] A. Léger et M. Pontier, *Drap brownien fractionnaire*, C. R. Acad. Sci. Paris Sér. I Math. 329 (1999), 893–898.
[15] B. Mandelbrot and J. Van Ness, *Fractional Brownian motions, fractional noises and applications*, SIAM Rev. 10 (1968), 422–437.
[16] I. Mendy and M. N'zi, *Approximation of fractional Brownian sheet by a random walk in anisotropic Besov space*, Random Oper. Stoch. Equ. 15 (2007), 137–154.
[17] D. Nualart, *The Malliavin Calculus and Related Topics*, 2nd ed., Springer, New York, 2006.
[18] A. S. Stoev and M. S. Taqqu, *How rich is the class of multifractional Brownian motions*, Stoch. Process. Appl. 116 (2006), 200–221.
[19] C. A. Tudor, *Weak convergence to the fractional Brownian sheet in Besov spaces*, Bull. Braz. Math. Soc. N. S. 34 (2003), 1–12.

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