KOLOKWIUM nr **5P**, **21.06.2022**, godz. 12:15–14:30

Zadania testowe (50 punktów – 1 punkt za każdą poprawną odpowiedź)

W każdym z zadań **1–20** podaj w **postaci uproszczonej** wartość całki oznaczonej.

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1. $\int\limits_{-2}^{10} |x|\,dx = \mathbf{52}$

2. $\int\limits_{-1}^{9} |x|\,dx = \mathbf{41}$

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3. $\int\limits_{0}^{10} |x-1|\,dx = \mathbf{41}$

4. $\int\limits_{0}^{10} |x-3|\,dx = \mathbf{29}$

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5. $\int\limits_{0}^{1} \sqrt{1-x^2}\,dx = \boldsymbol{\frac{\pi}{4}}$

6. $\int\limits_{0}^{2} \sqrt{4-x^2}\,dx = \boldsymbol{\pi}$

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7. $\int\limits_{0}^{2} \sqrt{8-x^2}\,dx = \boldsymbol{\pi+2}$

8. $\int\limits_{0}^{3} \sqrt{18-x^2}\,dx = \boldsymbol{\frac{9\pi}{4}+\frac{9}{2}}$

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9. $\int\limits_{0}^{4} \sqrt{32-x^2}\,dx = \boldsymbol{4\pi+8}$

10. $\int\limits_{0}^{5} \sqrt{50-x^2}\,dx = \boldsymbol{\frac{25\pi}{4}+\frac{25}{2}}$

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11. $\int\limits_{1}^{8} \frac{dx}{\sqrt[3]{x}} = \boldsymbol{\frac{9}{2}}$

12. $\int\limits_{1}^{16} \frac{dx}{\sqrt[4]{x}} = \boldsymbol{\frac{28}{3}}$

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13. $\int\limits_{9}^{14} \sqrt{x-5}\,dx = \boldsymbol{\frac{38}{3}}$

14. $\int\limits_{3}^{7} \sqrt{2x-5}\,dx = \boldsymbol{\frac{26}{3}}$

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15. $\int\limits_{2}^{4} \sqrt{4x-7}\,dx = \boldsymbol{\frac{13}{3}}$

16. $\int\limits_{1}^{2} \sqrt{8x-7}\,dx = \boldsymbol{\frac{13}{6}}$

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17. $\displaystyle\int_1^4 \frac{dx}{1+\sqrt{x}} = \mathbf{2+2\cdot\ln 2-2\cdot\ln 3}$

18. $\displaystyle\int_1^9 \frac{dx}{1+\sqrt{x}} = \mathbf{4-2\cdot\ln 2}$

19. $\displaystyle\int_4^{16} \frac{dx}{1+\sqrt{x}} = \mathbf{4+2\cdot\ln 3-2\cdot\ln 5}$

20. $\displaystyle\int_9^{16} \frac{dx}{1+\sqrt{x}} = \mathbf{2+4\cdot\ln 2-2\cdot\ln 5}$

W każdym z zadań **21–25** podaj w **postaci uproszczonej** promień zbieżności szeregu potęgowego.

21. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\cdot x^n}{n^n}$ $\qquad R = \mathbf{e}$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\cdot \binom{2n}{n}\cdot x^n}{n^n}$ $\qquad R = \mathbf{\frac{e}{4}}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\cdot \binom{2n}{n}\cdot x^{2n}}{n^n}$ $\qquad R = \mathbf{\frac{\sqrt{e}}{2}}$

24. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!\cdot x^{2n}}{n^n\cdot n!}$ $\qquad R = \mathbf{\frac{\sqrt{e}}{2}}$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{n!\cdot \binom{3n}{n}\cdot x^n}{n^n}$ $\qquad R = \mathbf{\frac{4e}{27}}$

W każdym z zadań **26–35** podaj w **postaci uproszczonej** normę supremum funkcji $f: D_f \to \mathbb{R}$ zdefiniowanej podanym wzorem na podanej dziedzinie.

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26. $f(x)=\dfrac{1}{x^2+10x+29}$ $D_f=\mathbb{R}$ $\|f\| = \mathbf{\dfrac{1}{4}}$

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27. $f(x)=\dfrac{1}{x^4+10x^2+31}$ $D_f=\mathbb{R}$ $\|f\| = \mathbf{\dfrac{1}{31}}$

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28. $f(x)=\dfrac{1}{x^8-10x^4+33}$ $D_f=\mathbb{R}$ $\|f\| = \mathbf{\dfrac{1}{8}}$

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29. $f(x)=\dfrac{1}{x^{12}+10x^6+37}$ $D_f=\mathbb{R}$ $\|f\| = \mathbf{\dfrac{1}{37}}$

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30. $f(x)=\dfrac{1}{x^{14}+10x^7+39}$ $D_f=\mathbb{R}$ $\|f\| = \mathbf{\dfrac{1}{14}}$

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31. $f(x)=2^x-7$ $D_f=(2,\,3)$ $\|f\| = \mathbf{3}$

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32. $f(x)=(2^x-7)^2-7$ $D_f=(2,\,3)$ $\|f\| = \mathbf{7}$

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33. $f(x)=(2^x-7)^2-17$ $D_f=(2,\,3)$ $\|f\| = \mathbf{17}$

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34. $f(x)=(2^x-7)^3+7$ $D_f=(2,\,3)$ $\|f\| = \mathbf{20}$

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35. $f(x)=(2^x-7)^3+17$ $D_f=(2,\,3)$ $\|f\| = \mathbf{18}$

W każdym z zadań **36–50** podaj sumę szeregu w postaci liczby całkowitej lub ułamka nieskracalnego.

Wiadomo, że

$$a_1=2, \quad a_2=1 \quad \text{oraz} \quad \sum_{n=1}^{\infty} a_n = 5\,.$$

Wobec tego:

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36. $\sum_{n=1}^{\infty} (a_n+a_{n+1}) = \mathbf{8}$ **37.** $\sum_{n=1}^{\infty} (a_n-a_{n+1}) = \mathbf{2}$

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38. $\sum_{n=1}^{\infty} \left(a_n^2-a_{n+1}^2\right) = \mathbf{4}$ **39.** $\sum_{n=1}^{\infty} \left(a_n^5-a_{n+1}^5\right) = \mathbf{32}$

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40. $\sum_{n=1}^{\infty} (a_n-a_{n+2}) = \mathbf{3}$ **41.** $\sum_{n=1}^{\infty} \left(a_n^3-a_{n+2}^3\right) = \mathbf{9}$

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42. $\sum_{n=1}^{\infty} \left(2^{a_n}-2^{a_{n+1}}\right) = \mathbf{3}$ **43.** $\sum_{n=1}^{\infty} \left(3^{a_n}-3^{a_{n+1}}\right) = \mathbf{8}$

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44. $\sum_{n=1}^{\infty} \left(2^{a_n}-2^{a_{n+2}}\right) = \mathbf{4}$ **45.** $\sum_{n=1}^{\infty} \left(3^{a_n}-3^{a_{n+2}}\right) = \mathbf{10}$

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46. $\sum_{n=1}^{\infty} \left(2^{2^{a_n}}-2^{2^{a_{n+1}}}\right) = \mathbf{14}$

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47. $\sum_{n=1}^{\infty} \left(3^{2^{a_n}}-3^{2^{a_{n+1}}}\right) = \mathbf{78}$

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48. $\sum_{n=1}^{\infty} \left(3^{2^{a_n}}-3^{2^{a_{n+2}}}\right) = \mathbf{84}$

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49. $\sum_{n=1}^{\infty} \left(\sqrt{1+a_n^3}-\sqrt{1+a_{n+1}^3}\right) = \mathbf{2}$

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50. $\sum_{n=1}^{\infty} \left(\sqrt{9+a_n^4}-\sqrt{9+a_{n+1}^4}\right) = \mathbf{2}$
