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July 1, 2024

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Report on the thesis
"On random walks in a sparse random environment"
by Alicia Kolodziejska

Dear Madams/Sirs,

it is a pleasure for me to give my evaluation of the PhD thesis "On random walks in a sparse random environment" submitted by Alicia Kolodziejska.

The thesis treats a one-dimensional stochastic process, called "Random Walks in a Sparse Random Environment" (RWSRE) which interpolates between simple symmetric random walk and random walk in an i.i.d. random environment. More precisely, the environment is random at the sites of a renewal process and in between these sites, the process behaves as a simple symmetric random walk. A nice feature of this model is that it exhibits different regimes, depending on the renewal process and the distribution of the environment at the sites of the renewal process. In particular, once can distinguish the moderately sparse regime (where the length of the interval between renewals has finite expectation) and the strongly sparse regime (where the latter expectation is infinite). As always for processes in random environment, there are two points of view: the "quenched" law fixes the environment whereas the "annealed" law averages over the environment.

The thesis is organized as follows.
It starts with a nice introduction, giving the definition of the model and highlighting the main results, as well as some of the notation used later. Chapter 2 discusses recurrence, transience and the speed of the process. These results are known already from previous papers about RWSRE. The results of the thesis are all for the case where the RWSRE is transient to the right. Moreover, formulas for the quenched expectations and variances of hitting times are derived in this chapter (see Lemma 2.3.1). Chapter 3 focuses on limit

theorems, both for the position of the walker and for hitting times. Theorem 3.2.1. gives, under a set of assumptions on renewal process and walk, a quenched central limit theorem for the position of the walker. The assumptions include a moment assumption on the interval between renewals, implying in particular that the process is in the moderately sparse regime. Chapter 4 studies limit laws in the strongly sparse regime. Theorem 4.2.2., Theorem 4.2.3. and Theorem 4.2.4. are among the main results of this thesis. Theorem 4.2.2. is particularly interesting since it demonstrates that - again under suitable assumptions on environment and renewal process - that the rescaled hitting times can not converge in distribution. It is not at all obvious how to prove such a result. An explanation and the proof (by contradiction) are given in Section 4.5. Chapter 5 gives annealed limit theorems for the maximal local times of RWSRE in the moderately sparse regime. The main results of this Chapter are Theorem 5.2.1, Theorem 5.2.2. and Theorem 5.2.3. An important tool is the associated branching process, nicely explained in Section 5.3.

Alicia Kolodziejska demonstrates with this thesis a deep understanding of the methods used in the field. She knows the tools needed in this area (renewal arguments, stable limit laws, branching processes, extreme value theory) and also knows how to apply and how to explain them.
The thesis is well-written - indeed, it is a pleasure to read it - and precise in the details. Many figures and explanations help the understanding. The thesis has led to the preprint "Weak quenched limit theorems for a random walk in a sparse random environment" in collaboration with Dariusz Buraczewski and Piotr Dyszewski, containing the main results of Chapter 4. Another preprint with the results of Chapter 5 is in preparation.

In short words, I full-heartedly recommend to accept this PhD thesis.

Sincerely,

Prof. Dr. Nina Gantert