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Report on the Doctoral Thesis "On boundaries of bicombable spaces" by Mr Daniel Danielski

15. Januar 2025

Dear Ladies and Gentlemen,

Following your request, I am pleased to present a report on the doctoral thesis "On boundaries of bicombable spaces", written by Mr Daniel Danielski.

The thesis consists of two largely separate chapters. The first chapter is about bicombings of metric spaces and groups acting on such spaces (in which case the bicombing is typically required to be equivariant). The author proves several foundational results, in most cases using the known results about Gromov-hyperbolic and CAT(0) spaces as motivations. The particular emphasis is about the study of the boundaries associated to equivariant bicombings.

After discussing some preliminaries in Section 1, In Sections 2 and 3 the author reviews two known constructions of the boundary associated to a $\mathfrak{ccc}$-bicombing - one using the geodesic rays compatible with a given $\mathfrak{ccc}$-bicombing, and another using an explicit C^*-algebra, and shows that these constructions are equivalent (Proposition 3.6). The boundary is used to obtain the first main result of the thesis - the construction of EZ-structures for groups which admit suitable actions on $\mathfrak{ccc}$-bicombed spaces.

Sections 4 and 5 examine whether the homeomorphism type of the boundary of a space with G-equivariant bicombing is an invariant of G. In particular, the author provides in Theorem 4.1 an example of a group G and two different geometric actions on bicombed spaces with the property that the boundaries are non-homeomorphic. The construction is motivated by examples which were used to study the analogous question in the CAT(0) case. The author states here several interesting open problems which arise from his results,

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both in the direction of obtaining the uniqueness of the homeomorphism type of the boundary for some special groups, as well as finding examples of groups with more than two different homeomorphism types of the boundary.

In Section 6 the author constructs a metric on the boundary of an equivariantly bicombed space and shows that the group action extends to an action by quasisymmetries on the boundary.

The topic of Section 7 are connections between the algebraic structure of a group G and some geometric properties of the action on the boundary. Although the results which are obtained are analogues of the properties known in the Gromov-hyperbolic and CAT(0) cases, the proofs require very substantial modifications in the case of general $\mathfrak{ccc}$-combings.

Section 8 is concerned with the property of being almost geodesically complete. This property is established (Theorem 8.1) for $\mathfrak{ccc}$-bicombed spaces with a suitable group action. The arguments are quite involved and go through establishing a link between this property and some results about non-vanishing of Alexander-Spanier cohomology groups. The results are motivated by similar results in the CAT(0) case.

The second, much shorter, chapter of the thesis is based on the joint article of the author, Michael Kapovich and Jacek Świątkowski. The main result is a complete characterisation of hyperbolic Coxeter groups whose Gromov boundary is homeomorphic to either the Sierpiński curve or the Menger curve. This work stemmed from the observation of Michael Kapovich that some previously known results from the paper [BK13] can be used to obtain such a complete characterisation. In this chapter all the technical details are provided which are needed to make this observation into an actual proof.

This thesis represents a significant original contribution to the field of mathematics and showcases author's superb acquaintance with current problems and methods related to a range of subjects of geometric group theory, and in particular boundaries of groups and spaces. Both the questions and the proofs are presented with a remarkable clarity, and the author shows an impressively good knowledge of the current state of the art, by providing extensive references to all relevant previous results and techniques. Taking all of this into account, I consider the work under review to be a very good doctoral thesis, and I wholeheartedly recommend awarding the author the doctoral degree.

Sincerely Yours,

Prof. Dr. Łukasz Grabowski