



Department of Mathematical Sciences

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Review of dissertation by Daniel Danielski

Mr. Danielski has written a nice dissertation on the topic of “boundaries at infinity” for certain spaces and groups. Most notably, he gives rigorous proofs that a group G which acts geometrically on a (finite-dimensional) metric space (X, d) that admits a G -equivariant \mathbf{ccc} bicombing, admits a corresponding EZ -structure. These EZ -structures are interesting in their own right. In addition, proofs of their existence contribute to what is known about the famous *Novikov Conjecture* in algebraic topology. Danielski’s work closes some gaps in the literature and unifies a pair of distinct approaches to obtaining EZ -structures. Unlike the case of hyperbolic groups, which have (essentially) unique boundaries, Danielski also shows that injective groups can admit more than one of his EZ -boundaries. The argument is delicate, and it reveals a lot about Danielski’s approach as it pertains to median geometries. As he notes, some interesting questions on that topic remain open. Several other interesting observations are made in the first part of this dissertation, such as the potentially useful fact that, when restricted to the Z -boundary, the G -actions occurring in this work are “by quasi-symmetry.”

The second part of this dissertation (joint with M. Kapovich and J. Swiatkowski) is focused on a single theorem. It gives a complete characterization of hyperbolic Coxeter groups whose boundary is a Sierpinski carpet or a Menger curve. Since these two spaces play an outsized role in the theory of hyperbolic group boundaries—and since Coxeter groups play an outsized role in geometric group theory—this is a notable theorem. On a personal note, my PhD student Cong He proved a related theorem for non-hyperbolic Coxeter groups in which he made significant use of related work by Danielski [Dan22]. So far, He’s work has not been posted on the arXiv, so I will attach an electronic copy of the dissertation to my email, in case the dissertator or other committee members are interested.

I was impressed by the clear, precise, and detailed exposition in this dissertation, and I congratulate Danielski on his work. There is nothing of great substance that I think needs to be revised. I did come across a few minor typographical errors, grammar issues, and other small problems. Rather than listing them in this report, I have marked them on a separate scanned pdf copy of the dissertation. I hope that will suffice.

Sincerely,

A handwritten signature in blue ink that reads "Craig R. Guilbault". The signature is written in a cursive, slightly slanted style.

Craig Guilbault