Abstract of the doctoral dissertation

The goal of the dissertation is to investigate various kinds of Riesz transforms on $\mathbb{R}^d$ with focus on obtaining dimension-free estimates of their L^p norms.

In the first part we handle classical Riesz transforms and maximal operators associated with them. We begin by using Fourier transform techniques to obtain a dimension-free estimate of the L^2 norm of the maximal Riesz transform in terms of the corresponding Riesz transform with an explicit constant. In order to accomplish this we factorize the maximal Riesz transform, following Mateu and Verdera, into the 'maximal part' and the 'Riesz part', namely

$$R_j^* = M^* R_j,$$

and estimate the Fourier multiplier associated with M^* in a dimension-free way.

Next, we use the real method of rotations and the complex method of rotations of Iwaniec and Martin to generalize this result to Riesz transforms of higher orders and to L^p norms for $1 < p < \infty$. We express the operator M^* as an integral of the Hilbert transform, thus obtaining a dimension-free estimate which is additionally explicit in terms of dependency on p.

In the second part we turn our attention to Riesz transforms related to Schrödinger operators, i.e. operators of the form

$$R_V^a = V^a L^{-a}, \qquad L = -\tfrac{1}{2}\Delta + V,$$

where Δ is the Laplacian, V is a non-negative potential, and L is called the Schrödinger operator. First we use complex interpolation to prove some general results on L^p-boundedness $(1 < p < \infty)$ of the operators R_V^a for locally integrable potentials. Then, using the Feynman–Kac formula and probabilistic methods we give conditions for the potential under which the operators R_V^a are bounded on L^1 and L^∞. In particular our results apply to potentials with power or exponential growth.

Finally, using similar methods, we show that if the potential V is of the form

$$V(x) = V_1(x) + \cdots + V_d(x),$$

where each V_i acts only on the i-th coordinate of the argument x and has polynomial growth with the exponent not greater than 2, then the L^1 and L^∞ norms of R_V^a can be estimated independently of the dimension. We achieve this by factorizing the semigroup associated with L into one-dimensional factors, estimating them separately and then combining the results.