Graph theory

Mock Class Test 2

The class test consists of two parts: "Exercises" and "Questions".
*Please attempt **BOTH** Exercises and **TWO** Questions.*

Refer to any results you are using by name
(or state them if you don't remember their name).

Duration of the class test: 90 minutes.

Exercises

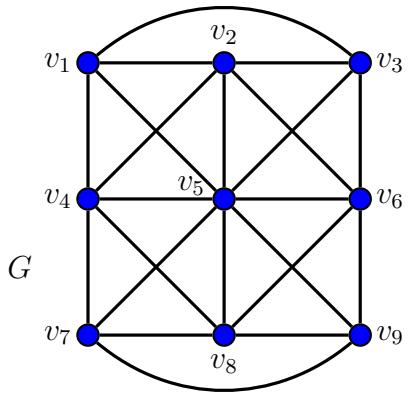
Please attempt ***BOTH*** *Exercises (A and B) displayed below.*

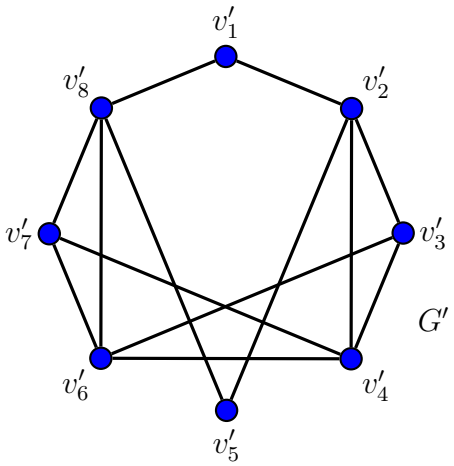
Wherever explanations are needed, please give precise *reasons (by explicitly writing down specific subgraphs, collections of vertices/edges, etc).*

The Exercises are worth 6 points in total.

Exercise A

Consider the following graphs G and G':





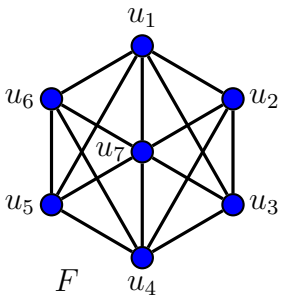
Decide whether or not each of these graphs is Hamiltonian. Explain your answers.

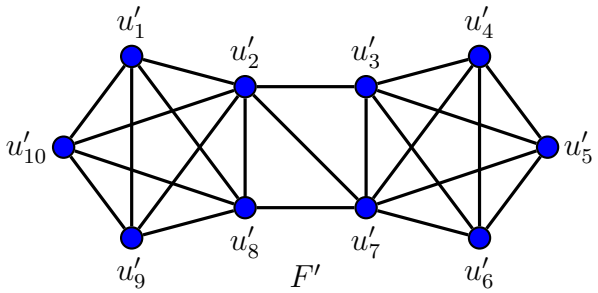
Solution for G:

Solution for G':

Exercise B

Consider the following graphs F and F':





Find the independence number α of each of these graphs, and explain your answers (giving reasons for both why the independence number is $\geq \alpha$ and why it is $\leq \alpha$).

Solution for F:

Solution for F':

Questions

*Please attempt **TWO** Questions (out of 3).*

Each Question is worth 7 points.

Question 1

Show that $R(C_4, C_4) = 6$.

Question 2

Let $n \geq 2$, and let G_n be a graph with $V(G_n) = \{(a, b) \in \mathbb{N}^2 \mid 1 \leq a < b \leq n\}$, such that $(a, b) \sim_{G_n} (a', b')$ if and only if either $a = b'$ or $b = a'$. By using the Multicolour Ramsey's Theorem, show that $\chi(G_n) \to \infty$ as $n \to \infty$.

Question 3

Let $p, \varepsilon \in (0, 1)$ be constants. Using the fact that $\ln(t!) = t \ln(t) - t + o(t)$, show that $\Delta(G) < (ep + \varepsilon)n$ for almost every $G \in \mathcal{G}(n, p)$ (where $e = 2.718\ldots$).