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EFFICIENT SEQUENTIAL PLANS FOR NONHOMOGENEOUS POISSON PROCESS

BY

ROMAN RÓŻAŃSKI (WROCŁAW)

Abstract. Consider a nonhomogeneous Poisson process with unknown intensity function $\lambda(s), s \geqslant 0$. The work answers the question: what are efficient sequential plans for this process? The efficiency is understanding in the sense of Cramer-Rao-Wolfowitz inequality.

Results obtained in this paper generalize theorems proved by Trybuła [7] for Poisson process with constant intensity.

1. CRAMER-RAO-WOLFOWITZ INEQUALITY AND WALD'S IDENTITIES FOR NONHOMOGENEOUS POISSON PROCESS

Let X_s, $s \geqslant 0$, be nonhomogeneous Poisson process with intensity function λ: $[0, \infty) \to [0, \infty)$, [2]. By $\mathscr{X}$ we denote the space of functions x: $[0, \infty) \to \mathscr{N}$; $\mathscr{N}$ – the set of nonnegative, integer numbers; constant in intervals and for which $x(0) = 0$, $x(s) = x(s-)+0 \vee 1$.

$\mathscr{B}$ is the smallest σ-algebra of subsets of $\mathscr{X}$, containing the sets $\{x \in \mathscr{X}: x(s) = k, s \geqslant 0, k \in \mathscr{N}\}$, $\mathscr{B}_t$ – the smallest σ-algebra containing the sets

$$\{x \in \mathscr{X}: x(s) = k, s \leqslant t, k \in \mathscr{N}\}.$$

Process X_s generates a measure μ_λ in the space $(\mathscr{X}, \mathscr{B})$, [3]. An unknown intensity function λ belongs to some function space A.

A Markov stopping time is a random variable τ: $\mathscr{X} \to [0, \infty]$ which satisfies the following conditions:

$$\{x \in \mathscr{X}: \tau(x) \leqslant t\} \in B_t, \forall t \geqslant 0;$$

$$\mu_\lambda(\{x \in \mathscr{X}: \tau(x) < \infty\}) = 1, \forall \lambda \in A.$$

A Markov stopping time τ generates a σ-algebra $\mathscr{B}_\tau$.

By μ_λ^τ we denote the measure μ_λ restricted to the σ-algebra $\mathscr{B}_\tau$.

We can formulate the following proposition, which is a consequence of theorem 19.7, [4].

Proposition 1. *We assume that*

$$\int_0^t \lambda(s)\,ds < \infty, \ \forall t \geqslant 0.$$

Let μ_1 *denote a measure generated by Poisson process with intensity equal to* 1. *If a Markov stopping time* τ *satisfies the condition*

$$\int_0^{\tau(x)} \left(1-\sqrt{\lambda(s)}\right)^2 ds < \infty \qquad \mu_\lambda\text{-almost surely},$$

then the measure μ_λ^τ *is absolutely continuous with respect to the measure* μ_1^τ *and*

$$\text{(1)} \qquad \frac{d\mu_\lambda^\tau}{d\mu_1^\tau}(x) = \exp\Big(\int_0^{\tau(x)} \ln\lambda(v)\,dx(v) + \int_0^{\tau(x)} [1-\lambda(v)]\,dv\Big)$$

$$= \begin{cases} \prod_{j=1}^{N_\tau(x)} \lambda(t_j)\exp\big(\tau - \int_0^\tau \lambda(v)\,dv\big) & \text{if } N_\tau > 0, \\ \exp\big(\tau - \int_0^\tau \lambda(v)\,dv\big) & \text{if } N_\tau = 0. \end{cases}$$

$N_\tau(x)$ *denotes a number of jumps of a realization* x *in the interval* $[0, \tau]$, $t_1, t_2, \ldots, t_{N_\tau}$ – *the times of jumps of a realization* x *in observed interval* $[0, \tau]$.

Proof. For any stopping time τ let us introduce stopped Poisson process $\tilde{X}_s = X_{s\wedge\tau}$. This process generates a measure $\tilde{\mu}_\lambda$ in the space $(\mathscr{X}, \tilde{\mathscr{B}})$, where $\tilde{\mathscr{B}}$ is the σ-algebra, generated by the sets

$$\{x\in\mathscr{X}\colon\ x(s\wedge\tau) = k,\ s \geqslant 0,\ k\in\mathscr{N}\}.$$

By theorem 6 [6] we have $\tilde{\mathscr{B}} = \mathscr{B}_\tau$ and $\mu_\lambda^\tau = \tilde{\mu}_\lambda$. The compensator A_t of the process X_t has the form

$$\int_0^t \lambda(s)\,ds.$$

So, from lemma 18.9 [4] we infer that the compensator $\tilde{A}_t$ of the process $\tilde{X}_t$ has the form $\tilde{A}_t = A_{t\wedge\tau}$.

Theorem 19.7 [4] allows us to conclude that $\mu_\lambda^\tau = \tilde{\mu}_\lambda \leqslant \tilde{\mu}_1 = \mu_1^\tau$ and taking $t = \infty$ we obtain formula (1).

Definition 1. A *sequential plan* is a pair $(\tau, f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau))$ where τ is a Markov stopping time and $f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau)$ is an estimator of the parameter $h(\lambda)h\colon A \to R$.

In the sequel, by $\nabla_\lambda g(\lambda)$ we denote a directional derivative at the point λ in the direction λ of the mapping g.

Now we can formulate the theorem about inequality of Cramer-Rao-Wolfowitz type.

THEOREM 1. *Let* $(\tau, f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau))$ *be a sequential plan for nonhomogeneous Poisson process with unknown intensity function, where* $f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau)$ *is unbiased estimator for the functional* $h(\lambda)$, *that means*

$$E_{\mu_\lambda} f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau) = h(\lambda)$$

and

$$\operatorname{Var}_{\mu_\lambda} f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau) < \infty .$$

We also assume that the function $d\mu_\lambda^\tau / d\mu_1^\tau$ *satisfies some regularity conditions, which guarante the following equations:*

$$\int_{\mathscr{X}} \nabla_\lambda \frac{d\mu_\lambda^\tau}{d\mu_1^\tau}(x, \lambda)\, d\mu_1(x) = 0, \tag{2}$$

$$\nabla_\lambda \int_{\mathscr{X}} f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau) \frac{d\mu_\lambda^\tau}{d\mu_1^\tau}(x, \lambda)\, d\mu_1(x) = \int_{\mathscr{X}} f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau) \nabla_\lambda \frac{d\mu_\lambda^\tau}{d\mu_1^\tau}(x, \lambda)\, d\mu_1(x). \tag{3}$$

Then

$$\operatorname{Var}_{\mu_\lambda} f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau) \geqslant \frac{[\nabla_\lambda h(\lambda)]^2}{\int_{\mathscr{X}} [\nabla_\lambda \ln \frac{d\mu_\lambda^\tau}{d\mu_1^\tau}(x, \lambda)]^2\, d\mu_\lambda(x)} \tag{4}$$

where

$$\nabla_\lambda \ln \frac{d\mu_\lambda^\tau}{d\mu_1^\tau}(x, \lambda) = N_\tau(x) - \int_0^\tau \lambda(v)\, dv. \tag{5}$$

The equality in (4) *holds at some* λ *if and only if*

$$f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau) = \frac{\nabla_\lambda h(\lambda)}{\int_{\mathscr{X}} [N_\tau(x) - \int_0^\tau \lambda(v)\, dv]^2\, d\mu_\lambda(x)} \left(N_\tau(x) - \int_0^\tau \lambda(v)\, dv\right) + h(\lambda) \tag{6}$$

μ_1*-almost surely.*

The proof of this theorem is analogous to that in [1] and [4].

Definition 2. A sequential plan $(\tau, f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau))$ is called an *efficient plan* if in formula (4) the equality holds for each $\lambda \in A$.

Let $\varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda)$ be a function μ_λ^τ-integrable. Moreover we suppose that

$$(7)\quad \nabla_\lambda \int_{\mathscr{X}} \varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda)\frac{d\mu_\lambda^\tau}{d\mu_1^\tau} d\mu_1^\tau = \int_{\mathscr{X}} \nabla_\lambda \left[\varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda)\frac{d\mu_\lambda^\tau}{d\mu_1^\tau}\right] d\mu_1^\tau.$$

So we can write:

$$(8)\quad E_{\mu_\lambda}[N_\tau - \int_0^\tau \lambda(v)\,dv]\,\varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda) = \nabla_\lambda E_{\mu_\lambda} \varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda) - E_{\mu_\lambda} \nabla_\lambda \varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda).$$

If we put $\varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda) = 1$ in formula (8), we obtain the first Wald identity:

$$(9)\quad E_{\mu_\lambda} N_\tau = E_{\mu_\lambda} \int_0^\tau \lambda(v)\,dv.$$

If we put

$$\varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda) = N_\tau - \int_0^\tau \lambda(v)\,dv,$$

then from (8) we obtain the second Wald identity:

$$(10)\quad E_{\mu_\lambda}[N_\tau - \int_0^\tau \lambda(v)\,dv]^2 = E_{\mu_\lambda} \int_0^\tau \lambda(v)\,dv.$$

Putting $\varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda) = N_\tau$ we obtain:

$$(11)\quad E_{\mu_\lambda}[N_\tau - \int_0^\tau \lambda(v)\,dv]\,N_\tau = \nabla_\lambda E_{\mu_\lambda} \int_0^\tau \lambda(v)\,dv.$$

Now let

$$\varphi(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau, \lambda) = \int_0^\tau \lambda(v)\,dv.$$

Then we have

$$(12)\quad E_{\mu_\lambda}[N_\tau - \int_0^\tau \lambda(v)\,dv] \int_0^\tau \lambda(v)\,dv = \nabla_\lambda E_{\mu_\lambda} N_\tau - E_{\mu_\lambda} \int_0^\tau \lambda(v)\,dv.$$

We can write

$$(13)\quad \mathrm{Var}_{\mu_\lambda} N_\tau = \mathrm{Var}_{\mu_\lambda}[\int_0^\tau \lambda(v)\,dv] + 2\nabla_\lambda E_{\mu_\lambda} \int_0^\tau \lambda(v)\,dv - E_{\mu_\lambda} \int_0^\tau \lambda(v)\,dv.$$

2. EFFICIENCY OF A FIXED-TIME PLAN

Definition 3. A sequental plan (τ, f), where τ is equal, with probability 1, to a constant $t > 0$, is called a *fixed-time plan.*

THEOREM 2. *If some regularity conditions, which guarantee equalities* (2), (3), *and* (7), *are satisfied, then a fixed-time plan is efficient.*

Proof. Let

$$f(\tau, t_1, t_2, \ldots, t_{N_\tau}, N_\tau) = f(t, N_t) = aN_t + b,$$

$$E_{\mu_\lambda} f(t, N_t) = h(\lambda) = a\int_0^t \lambda(v)\,dv + b,$$

$$\mathrm{Var}_{\mu_\lambda} f(t, N_t) = a^2 \int_0^t \lambda(v)\,dv.$$

The lower bound in the Cramer-Rao-Wolfowitz inequality takes the following form:

$$\frac{[\nabla_\lambda h(\lambda)]^2}{E_{\mu_\lambda}\left[\nabla_\lambda \ln \frac{d\mu_\lambda^\tau}{d\mu_1^\tau}\right]^2} = \frac{a^2 [\int_0^t \lambda(v)\,dv]^2}{\int_0^t \lambda(v)\,dv} = a^2 \int_0^t \lambda(v)\,dv.$$

So, a fixed-time plan is efficient sequential plan and

$$h(\lambda) = a\int_0^t \lambda(v)\,dv + b$$

is efficiently estimable functional of λ for this plan.

The estimator $f(t, N_t) = aN_t + b$ is efficient estimator for a fixed-time plan.

Remark. If intensity function $\lambda \in C[0, t]$, then, from theorem XII 20′ [5], equalities (2), (3), (8)-(13) hold for a fixed-time plan.

3. EFFICIENCY OF AN OBLIQUE PLAN

Definition 4. A sequential plan (τ_u, f), where

$$\tau_u = \inf\left\{t\colon\ N_t = \frac{1}{r}(t-s)\right\}, \qquad r > 0,\ s > 0,$$

with probability 1, is called an *oblique plan.*

In the sequel we assume that λ is continuous, periodic function with the period equal to r.

3.1. Existing and finiteness of the first two moments of an oblique plan. Let $p_i(s)$ denote a probability of the first attaining of the line $k = (t-s)/r$ at the point i, by the process N_t.

We can write the following equality:

$$\frac{p_i(s+\Delta s)-p_i(s)}{\Delta s} = -\frac{1-\exp(-\int_s^{s+\Delta s}\lambda(v)\,dv)}{\Delta s}\,p_i(s)+$$

$$+\frac{(\int_s^{s+\Delta s}\lambda(v)\,dv)\cdot\exp(-\int_s^{s+\Delta s}\lambda(v)\,dv)}{\Delta s}\times$$

$$\times\Big\{p_{i-1}(s)\exp(-\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)+p_{i-2}(s)\times$$

$$\times\Big[(\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)\exp(-\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)\exp(-\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)+$$

$$+\frac{1}{2}(\int_s^{s+\Delta s}\lambda(v)\,dv)\exp(-\int_{s+\Delta s}^{s+\Delta s+2r}\lambda(v)\,dv)\Big]+$$

$$+p_{i-3}(s)\Big[(\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)^2\exp(-3\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)+\frac{1}{2}(\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)^2\times$$

$$\times\exp(-\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)\exp(-\int_{s+\Delta s}^{s+\Delta s+2r}\lambda(v)\,dv)+\frac{1}{2}(\int_s^{s+\Delta s}\lambda(v)\,dv)\times$$

$$\times(\int_{s+\Delta s}^{s+\Delta s+2r}\lambda(v)\,dv)\exp(-\int_{s+\Delta s}^{s+\Delta s+2r}\lambda(v)\,dv)\exp(-\int_{s+\Delta s}^{s+\Delta s+r}\lambda(v)\,dv)+$$

$$+\frac{1}{3!}(\int_s^{s+\Delta s}\lambda(v)\,dv)^2\exp(-\int_{s+\Delta s}^{s+\Delta s+3r}\lambda(v)\,dv)\Big]+\ldots+p_0(s)L\Big\},$$

where

$$L(\int_s^{s+\Delta s}\lambda(v)\,dv)\exp(-\int_s^{s+\Delta s}\lambda(v)\,dv)$$

denotes the probability of the first attaining of the line $(t-(s+\Delta s))/r$ at the point i after first attaining of the line $(t-s)/r$ at the point 0. If $\Delta s \to 0$ we obtain:

$$p_i'(s) = -\lambda(s)\,p_i(s)+\lambda(s)\,p_{i-1}(s+r), \tag{14}$$

$$p_0'(s) = -\lambda(s)\,p_0(s),$$

$$p_0(0) = 1, \quad p_i(0) = 0 \quad \text{for } i \neq 0.$$

We have

$$p_0(s) = \exp\left(-\int_0^s \lambda(v)\,dv\right).$$

We seek solution of the form

$$p_i(s, \lambda) = q_i(s, \lambda)\exp\left(-\int_0^{s+ir} \lambda(v)\,dv\right).$$

Then we obtain the following system of equalities:

$$q_i'(s, \lambda) = \lambda(s)\,q_{i-1}(s+r), \tag{15}$$

$$q_0(0) = 1, \quad q_i(0) = 0 \quad \text{for } i \neq 0.$$

This system of equations has the following solution:

$$q_i(s) = \frac{\int_0^s \lambda(v)\,dv}{i!}\left(\int_0^s \lambda(v)\,dv + i\int_0^r \lambda(v)\,dv\right)^{i-1},$$

$$q_0(s) = 1.$$

So the solution of (14) has the form

$$p_i(s, \lambda) = \frac{1}{i!}\left(\int_0^s \lambda(v)\,dv\right)\left(\int_0^s \lambda(v)\,dv + i\int_0^r \lambda(v)\,dv\right)^{i-1} \times \exp\left[-\left(\int_0^s \lambda(v)\,dv + i\int_0^r \lambda(v)\,dv\right)\right], \tag{16}$$

$$p_0(s, \lambda) = \exp\left(-\int_0^s \lambda(v)\,dv\right).$$

THEOREM 3. *If* $\int_0^r \lambda(v)\,dv < 1$, *then* $\sum_i p_i(s, \lambda) = 1$.

Proof. We have

$$\mu_\lambda(\tau_u \geqslant t) \leqslant \mu_\lambda\left(\left|x_t - \int_0^t \lambda(v)\,dv\right| \geqslant \frac{1}{r}(t-s) - \int_0^t \lambda(v)\,dv\right)$$

$$\leqslant \frac{\int_0^t \lambda(v)\,dv}{\left[\frac{1}{r}(t-s) - \int_0^t \lambda(v)\,dv\right]^2}$$

for sufficiently large t under the assumption that

$$\int_0^r \lambda(v)\,dv < 1.$$

But

$$\lim_{t\to\infty} \frac{\int_0^t \lambda(v)\,dv}{\left[\frac{1}{r}(t-s)-\int_0^t \lambda(v)\,dv\right]^2} = 0,$$

$$\mu_\lambda(\tau_u = \infty) \leqslant \lim_{t\to\infty} \mu_\lambda(\tau_u \geqslant t) = 0,$$

$$\sum_i p_i(s, \lambda) = \mu_\lambda(\tau_u < \infty) = 1.$$

THERthe 4. *If the intensity function λ is a continuous periodic function with the period equal to r and*

$$\int_0^r \lambda(v)\,dv < 1,$$

then

(17) $$E_{\mu_\lambda} N_{\tau_u} = M(s) = \frac{\int_0^s \lambda(v)\,dv}{1-\int_0^r \lambda(v)\,dv}.$$

Proof. We have $\sum_i p_i(s, \lambda) = 1$.

Theorem XII 20′ [5] allows us to go with directional derivative, with respect to λ, under the sum sign. We obtain

$$\sum_i ip_i(s)-\Big(\int_0^s \lambda(v)\,dv\Big)\sum_i p_i(s)-\Big(\int_0^r \lambda(v)\,dv\Big)\sum_i ip_i(s) = 0,$$

$$M(s)-\int_0^s \lambda(v)\,dv-\Big(\int_0^r \lambda(v)\,dv\Big)M(s) = 0.$$

So we obtain formula (17).

THEOREM 5. *If the intensity function λ is a continuous periodic function with the period r and*

$$\int_0^r \lambda(v)\,dv < 1,$$

then

(18) $$E_{\mu_\lambda} N_{\tau_\mu}^2 = K(s) = \frac{\Big(\int_0^s \lambda(v)\,dv\Big)^2-\Big(\int_0^s \lambda(v)\,dv\Big)^2\Big(\int_0^r \lambda(v)\,dv\Big)+\int_0^s \lambda(v)\,dv}{\Big(1-\int_0^r \lambda(v)\,dv\Big)^3},$$

(19)
$$\operatorname{Var}_{\mu_\lambda} N_{\tau_u} = \frac{\int_0^s \lambda(v)\,dv}{\left(1-\int_0^r \lambda(v)\,dv\right)^3}.$$

Proof. Let us consider previously proved equality:

$$\sum_i ip_i(s) - \int_0^s \lambda(v)\,dv - \left(\int_0^r \lambda(v)\,dv\right)\sum_i ip_i(s) = 0.$$

Under the assumptions about the intensity function λ, we can use theorem XII 20′ [5] and go with the directional derivative, with respect to λ, under the sum sign. We obtain:

$$K(s)\left(1-\int_0^r \lambda(v)\,dv\right)^2 - \left(\int_0^s \lambda(v)\,dv\right)M(s) -$$
$$-\left(\int_0^r \lambda(v)\,dv\right)M(s) + \left(\int_0^r \lambda(v)\,dv\right)\left(\int_0^s \lambda(v)\,dv\right)M(s) - \int_0^s \lambda(v)\,dv = 0.$$

Using formula (17) for $M(s)$ we can obtain formulas (18) and (19).

3.2. Efficiency of an oblique plan.

THEOREM 6. *If the intensity function λ is a continuous periodic function with the period r and*

$$\int_0^r \lambda(v)\,dv < 1,$$

then the oblique plan is an efficient sequential plan.

Proof. By theorem XII 20′ [5] we infer that for an oblique plan the regularity conditions, guaranteeing equalities (2), (3), (9)-(13), hold. For an oblique plan we can write

$$N_{\tau_u} = \frac{1}{r}(\tau_u - s).$$

Let

$$f(t_1, t_2, \ldots, t_{N_{\tau_u}}, N_{\tau_u}, \tau_u) = a\tau_u + b,$$

$$h(\lambda) = a\frac{r\int_0^s \lambda(v)\,dv - s\int_0^r \lambda(v)\,dv + s}{1-\int_0^r \lambda(v)\,dv} + b,$$

$$\mathrm{Var}_{\mu_\lambda} f = a^2 \frac{r^2 \int_0^s \lambda(v)\,dv}{(1-\int_0^r \lambda(v)\,dv)^3},$$

$$\frac{(\nabla_\lambda h(\lambda))^2}{E_{\mu_\lambda}[N_{\tau_u} - \int_0^{\tau_u} \lambda(v)\,dv]^2} = a^2 \frac{r^2 \int_0^s \lambda(v)\,dv}{(1-\int_0^r \lambda(v)\,dv)^3}.$$

So an oblique plan is efficient plan.

4. EFFICIENCY OF AN INVERSE PLAN

Definition 5. A sequential plan (τ_0, f), where

$$\tau_0(x) = \inf\{t:\ N_t(x) = l_0\}$$

with probability 1, is called an *inverse plan.*

We can write the following formula for the density function $g_{\tau_0}(t)$ of the stopping time τ_0:

$$g_{\tau_0}(t) = \frac{\lambda(t)}{(l_0-1)!}\Big(\int_0^t \lambda(v)\,dv\Big)^{l_0-1} \exp\Big(-\int_0^t \lambda(v)\,dv\Big).$$

Let us consider the estimator

$$f(\tau_0, t_1, t_2, \ldots, t_{N_{\tau_0}}, N_{\tau_0}) = a\tau_0 + b.$$

The lower bound in the Cramer-Rao-Wolfowitz inequality takes the following form:

$$\frac{a^2}{l_0[(l_0-1)!]^2}\Big[\int_0^\infty \Big(\int_0^t \lambda(v)\,d\lambda\Big)^{l_0} \exp\Big(-\int_0^t \lambda(v)\,dv\Big)dt\Big]^2.$$

Taking $l_0 = 1$ we can check that an inverse plan is not efficient one.

We can conclude that an inverse plan is not efficient one for each l_0 and possibly wide class of intensity functions containing constant functions. But, as is proved in [7], an inverse plan is a complete plan.

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Institute of Mathematics
Technical University of Wrocław
Wybrzeże Wyspiańskiego 27
50-370 Wrocław, Poland

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