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THE LADDER VARIABLES OF A MARKOV RANDOM WALK

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Abstract: Given a Harris chain $(M_n)_{n\geq 0}$ on any state space $(\mathcal{S},\mathcal{C})$ with essentially unique stationary measure ξ, let $(X_n)_{n\geq 0}$ be a sequence of real-valued random variables which are conditionally independent, given $(M_n)_{n\geq 0}$, and satisfy

$$P(X_k \in \cdot|(M_n)_{n\geq 0}) = Q(M_{k-1}, M_k, \cdot)$$

for some stochastic kernel $Q : \mathcal{S}^2 \times \mathcal{B} \to [0,1]$ and all $k \geq 1$. Denote by S_n the n-th partial sum of this sequence. Then $(M_n, S_n)_{n\geq 0}$ forms a so-called Markov random walk with driving chain $(M_n)_{n\geq 0}$. Its stationary mean drift is given by $\mu = E_\xi X_1$ and assumed to be positive in which case the associated (strictly ascending) ladder epochs

$$\sigma_0 = \inf\{k \geq 0 : S_k \geq 0\},$$

$$\sigma_n = \inf\{k > \sigma_{n-1} : S_k > S_{\sigma_{n-1}}\} \quad \text{for } n \geq 1,$$

and the ladder heights $S_n^* = S_{\sigma_n}$ for $n \geq 0$ are a.s. positive and finite random variables. Put $M_n^* = M_{\sigma_n}$. The main result of this paper is that $(M_n^*, S_n^*)_{n\geq 0}$ and $(M_n^*, \sigma_n)_{n\geq 0}$ are again Markov random walks (with positive increments, thus so-called Markov renewal processes) with Harris recurrent driving chain $(M_n^*)_{n\geq 0}$. The difficult part is to verify the Harris recurrence of $(M_n^*)_{n\geq 0}$. Denoting by ξ^* its stationary measure, we also give necessary and sufficient conditions for the finiteness of $E_{\xi^*} S_1^*$, $E_\xi S_1^*$ and $E_{\xi^*}\sigma_1$ in terms of μ or the recurrence-type of $(M_n)_{n\geq 0}$ or $(M_n^*)_{n\geq 0}$.

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