Integration of semialgebraic functions and antiderivatives of Nash functions

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Abstract: When one integrates semialgebraic functions (with parameters) one has to leave the semialgebraic setting. For example, one gets the global logarithm as the antiderivative of $1/x$ and (iterated) antiderivatives of algebraic power series as $\arctan(x)$. We show that it is enough to enlarge the semialgebraic functions by these functions to obtain a complete picture of the situation.

To realize this we close the rings of algebraic power series in arbitrary dimension under taking antiderivatives. We analyze these rings profoundly. In particular we show that the Weierstraß division theorem and the Weierstraß preparation theorem hold. This allows us to apply deep results from model theory to describe integrals of semialgebraic functions explicitly.