A partial order $\mathbb{P}$

- has Knaster Property, if every uncountable family $\mathcal{F} \subseteq \mathbb{P}$ contains an uncountable subfamily $\mathcal{G} \subseteq \mathcal{F}$ which is linked, i.e. A and B are compatible for each $A, B \in \mathcal{G}$;
- is σ-centered, if $\mathbb{P} = \bigcup_{n\in\omega} \mathcal{C}_n$, where each $\mathcal{C}_n$ is centered;
- has a countable π-base, if there is a countable $\mathcal{P} \subseteq \mathbb{P}$ such that for every $p \in \mathbb{P}$ there is $p' \in \mathcal{P}$ such that $p' \leq p$.

(We assume here that $\mathbb{P}$ does not have the smallest element.) We say that a topological space K has Knaster Property (or has countable π-base) if the family of its nonempty open subsets (with inclusion) has Knaster Property (countable π-base).

Δ-Lemma. Every uncountable family of finite sets has an uncountable subfamily $\mathcal{A}$ which is a Δ system, i.e. is such that there is (a root) R such that for distinct $A, B \in \mathcal{A}$ we have $A \cap B = R$

Zad. 1 Show that the existence of countable π-base implies separability which itself implies Knaster property and that Knaster property implies ccc.

Zad. 2 Let $\kappa > \omega$. Show that 2^κ is ccc. Does it have Knaster property? Show that it does not have a countable π-base. (Hint: Δ-lemma.)

Zad. 3 Show that if K supports a strictly positive measure μ (i.e. μ is positive on nonempty open sets), then K has Knaster property (Hint: use Duschnik-Miller theorem).

Zad. 4 Show that amoeba forcing is a poset with Knaster property which is not σ-centered.

Zad. 5 Show that if there exists a Suslin tree, then it exists a tall Suslin tree. (Suslin tree $\mathbb{S}$ is tall if for each $s \in \mathbb{S}$ and each each $\alpha < \omega_1$ there is $t \in \mathbb{S}$ which is comparable with s.)

Zad. 6 Show that Suslin tree is not σ-centered.

Zad. 7 Let $\mathbb{S}$ be Suslin tree and let $\mathfrak{A}$ be the Boolean algebra generated by $\mathbb{S}$. Show that for every $A \in \mathfrak{A}$ there is $S \in \mathbb{S}$ such that $S \leq A$. Conclude, that $\mathfrak{A}$ is ccc.