Applications of infinitary combinatorics 7 2018

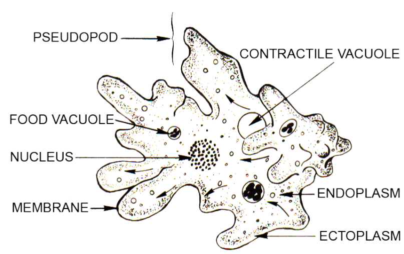
Zad. 1 (Amoeba forcing)

- Let $\varepsilon > 0$. Show that the poset

$$\mathbb{A}_\varepsilon = \{U \subseteq \mathbb{R} \colon \lambda(U) < \varepsilon, U \text{ - open}\}$$

  ordered by $\supseteq$ is ccc. (Hint: consider an uncountable family $\mathcal{R} \subseteq \mathbb{A}_\varepsilon$. For each $R \in \mathcal{R}$ fix a finite union of rational intervals (i.e. intervals with rational endpoints) $R' \subseteq R$ such that $\lambda(R - R') < \varepsilon - \lambda(R)$. Use the fact that there are only countably many such unions.)

- Show that if $N \in \mathcal{N}$, then $D_N = \{U \in \mathbb{A}_\varepsilon \colon N \subseteq U\}$ is dense in $\mathbb{A}_\varepsilon$.
- Show that if G is generic over $\mathbb{A}_\varepsilon$, then $\lambda(\bigcup G) \leq \varepsilon$.
- Conclude that MA implies that $\mathrm{add}(\mathcal{N}) = \mathfrak{c}$ (and so that all coefficients in Cichon's diagram equal $\mathfrak{c}$).



Zad. 2 Let $\mathbb{M}$ be the following ideal on $\mathbb{R}^2$:

$$\mathbb{M} = \{M \subseteq \mathbb{R}^2 \colon \exists B \in \mathrm{Borel}(\mathbb{R}^2)\ M \subseteq B \text{ and } \forall x \in \mathbb{R}\ \lambda(B_x) = 0\}.$$

What you can say about non and cov for this ideal? About add ($\lightning$)?

Zad. 3 A set $L \subseteq \mathbb{R}$ (of size $\mathfrak{c}$) is called *generalized Luzin set*, if $|L \cap M| < \mathfrak{c}$ for each $M \in \mathcal{M}$. Show that if $\mathrm{add}(\mathcal{M}) = \mathfrak{c}$, then there is a Luzin set.

Zad. 4 (Erdosa-Ulama theorem.) Show that if $\mathrm{add}(\mathcal{N}) = \mathrm{cof}(\mathcal{N})$, then there is a bijection $f\colon \mathbb{R} \to \mathbb{R}$ such that $f(A) \in \mathcal{M} \iff A \in \mathcal{N}$.

Hint: Consider an increasing base $(N_\alpha)_{\alpha<\kappa}$ of the ideal $\mathcal{N}$ and an increasing base $(M_\alpha)_{\alpha<\kappa}$ of the ideal $\mathcal{M}$ (where $\kappa = \mathrm{add}(\mathcal{N})$). Assume that $N_0 \cup M_0 = \mathbb{R}$ and $N_0 \cap M_0 = \emptyset$ and that $|N_{\alpha+1} \setminus N_\alpha| = \mathfrak{c}$ (similarly for $(M_\alpha)_{\alpha<\kappa}$). Define the bijection on $N_{\alpha+1} \setminus N_\alpha \ldots$

Zad. 5 We say that $(x, (I_n))$ is a *chopped real* if $x \in \omega^\omega$ and (I_n) is an interval partition of ω. Say that a real $y \in \omega^\omega$ *matches* $(x, (I_n))$ if there are infinitely many n's such that $y_{|I_n} = x_{|I_n}$. Show that $M\mathcal{M}$ iff there is $(x, (I_n))$ such that no $y \in M$ matches $(x, (I_n))$.

Zad. 6 Consider the following orderings: 1, ω, ω_1, $\omega \times \omega_1$, $[\omega_1]^{<\omega}$. What are the Tukey relations between those partial orders? Can you find a partial order of size at most ω_1 which is not Tukey equivalent to any of them? ($\mathbb{P}$ is Tukey equivalent to $\mathbb{Q}$ iff $\mathbb{P}$ is Tukey below $\mathbb{Q}$ and $\mathbb{Q}$ is Tukey below $\mathbb{P}$).

Pbn
`http://www.math.uni.wroc.pl/~pborod/dydaktyka`