Applications of infinitary combinatorics 6 2018

Zad. 1 Show that $(\beta\omega, +)$ is left topological semi-group.

Zad. 2 Show that $(m) + p = p + (m)$ for every $p \in \beta\omega$ and $m \in \omega$.

Zad. 3 Show that if $p \in \beta\omega \setminus \omega$ extends the filter of sets with density 1, then for each q the ultrafilter $p + q$ extends the filter of sets with density 1. Prove that if $p \in \beta\omega \setminus \omega$ contains a density 0 set then for each q the ultrafilter $p + q$ contains a set with density 0. Conclude that the addition of ultrafilters is not commutative. Also, that the function T_q defined by $T_q(p) = p + q$ is not continuous.

Zad. 4 Show that if p if P-point, then p is not of the form $p = q + r$ for any q, $r \in \beta\omega \setminus \omega$.

Zad. 5 Let $(G, +)$ be a compact left topological semi-group. Show that every right ideal contains a minimal right ideal and that this minimal ideal is closed.

Zad. 6 Show that if I is an ideal in a semi-group $(G, +)$ and R is a minimal right ideal, then $R \subseteq I$.

Zad. 7 Fix $k \in \omega$ and work in $(\beta\omega)^k$. Let

$$S = \{(n, n + d, n + 2d, \ldots, n + (k-1)d) \colon d, n \in \omega\}$$

and

$$I = \{(n, n + d, n + 2d, \ldots, n + (k-1)d) \colon n \in \omega, d > 0\}.$$

Show that $\overline{S}$ is a left topological semi-group and that $\overline{I}$ is an ideal in $\overline{S}$.

Zad. 8 Let Σ be a finite alphabet and let W be the set of words over Σ. Let v be a letter outside Σ (a *variable* over W), A - the set of words over $\Sigma \cup \{v\}$ and $V = A \setminus W$. For every $a \in \Sigma$ let $\bar{a} \colon A \to W$ be a function such that $\bar{a}(w)$ is a word in Σ in which all instances of v in w are replaced by a. Prove the following theorem: for every finite partition of W there is $x \in V$ and an element of the partition such that $\bar{a}(x)$ is contained in this element for each $a \in A$. (This is Hales-Jewett theorem. See the Blass' paper linked on the webpage).

Pbn
`http://www.math.uni.wroc.pl/~pborod/dydaktyka`