## Applications of infinitary combinatorics 5 2018

**Zad. 1** Define the *almost disjointness number* $\mathfrak{a}$ by

$$\mathfrak{a} = \min\{|\mathcal{A}| \colon \mathcal{A} \text{ is infinite maximal almost disjoint family}\}.$$

Show that $\mathfrak{b} \leq \mathfrak{a}$. Hint: consider $\mathcal{A}$ - almost disjoint family of size less than $\mathfrak{b}$ on $\omega \times \omega$ and suppose that it contains all the sets of the form $\{n\} \times \omega$.

**Zad. 2** What is the topological interpretation of $\mathfrak{a}$?

**Zad. 3** We say that $S \subseteq \omega$ splits $N \subseteq \omega$ if both $S \cap A$ and $A \setminus S$ are infinite. Define the *splitting number*

$$\mathfrak{s} = \min\{|\mathcal{A}| \colon \text{ every } N \subseteq \omega \text{ is split by a member of } \mathcal{A}\}.$$

Show that $\mathfrak{s}$ is the minimal family of bounded sequences of real numbers such that for every infinite $Y \subseteq \omega$ at least one of those sequences does not converge on $Y$.

**Zad. 4** Show that $\mathfrak{s}$ is the minimal $\kappa$ such that $\{0,1\}^\kappa$ is sequentially compact (i.e. every sequence has a convergent subsequence).

**Zad. 5** Show that if $X \subseteq \{0,1\}^\omega$ is such that $|X| < \mathfrak{s}$, then $\lambda(X) = 0$, where $\lambda$ is the standard (Lebesgue) measure on $\{0,1\}^{\mathbb{N}}$.

**Zad. 6** Show directly (without using $\mathfrak{p} \leq \mathfrak{a}$) that $\mathsf{MA}(\omega_1)$ implies $\mathfrak{a} > \omega_1$.

**Zad. 7** Assume $\mathsf{MA}(\kappa)$. Let $X$ be a Hausdorff space such that every family of pairwise disjoint open sets is countable. Let $\{U_\alpha \colon \alpha < \kappa\}$ be a family of dense open subsets of $X$. Show that $\bigcap_{\alpha<\kappa} U_\alpha \neq \emptyset$. Notice that this is a generalization of Baire theorem.

**Zad. 8** Consider the following version of Martin's Axiom: for every $\mathbb{P}$ - **countable** partially ordered set, for every family of $\kappa$ many dense sets in $\mathbb{P}$, there is a filter intersecting all of them. Show that this version implies that $\mathfrak{d} = \mathfrak{c}$. Hint: consider

$$\mathbb{P} = \{f \colon f - \text{ function}, \mathrm{dom}(f) \in [\omega]^{<\omega}, \mathrm{rng}(f) \subseteq \omega\}$$

ordered by "$\supseteq$".

**Zad. 9** Show that $\mathfrak{b}$ is the minimal number $\kappa$ such that there is no gap of type $(\omega, \kappa)$. What is the topological interpretation of this fact?

Pbn
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