Applications of infinitary combinatorics 1 2018

Basic structures in $\mathcal{P}(\omega)/Fin$.

Zad. 1 Show that there is a maximal almost disjoint family of size $\mathfrak{c}$ of subsets of $\mathbb{Z}\times\mathbb{Z}$ parametrized by lines on $\mathbb{R}^2$ going through the origin. (That was the third, unfinished, proof of the existence of MAD of size $\mathfrak{c}$, which I presented on the lecture.)

Zad. 2 Show there is no ω-gap. It means: assume $(L_\alpha)_{\alpha<\omega}$ and $(R_\alpha)_{\alpha<\omega}$ are families of subsets of ω such that

- $L_\alpha \subseteq^* L_\beta$ for each $\alpha < \beta$,
- $R_\beta \subseteq^* R_\alpha$ for each $\alpha < \beta$,
- $L_\alpha \subseteq^* R_\beta$ for each α, β.

Show that there is $L \subseteq \omega$ such that $L_\alpha \subseteq^* L$ and $L^* \subseteq R_\alpha$ for every $\alpha < \omega$.

Zad. 3 Show an example of an uncountable independent family of subsets of $\{0,1\}^{\omega_1}$.

Zad. 4 Show that each Mrówka space is first countable (i.e. every point is G_δ) and locally compact. When Mrówka space is compact? (Here: `http://www.matmor.unam.mx/~michael/preprints_files/Mrowkaspaces.pdf` you can see how many properties can be studied in the context of Mrówka spaces.)

Zad. 5 Show that every normal pseudocompact space is countably compact. Hint: show that if X is not countably compact, then it contains a countable discrete closed subspace. Then define an appropriate function on this subspace and use Tietze theorem.

Zad. 6 There is a sequence of dwarfs enumerated by natural numbers staying in a way that 0 sees all the other dwarfs, 1 sees $\omega \setminus \{0,1\}$, 2 sees $\omega \setminus \{0,1,2\}$ and so on. Somebody put on their heads green and red hats. Each dwarf has to guess what is the color of his hat. If he does not guess it right, we will naturally cut his head.

Version 1. We are asking dwarfs about the colors, starting from 0. So, n hears the answers of 0, 1, ..., $n-1$.

Version 2. We are asking all the dwarfs at once.

Dwarf are allowed to discuss a strategy before standing in the sequence. Try to invent a strategy for dwarfs, allowing them to save as many dwarfs lives as possible. How many dwarfs we can save in Version 1? How many in Version 2? (Remark: dwarfs cannot say anything but *red* or *green*).

Zad. 7 100 mathematicians are supposed to go to prison, to the separate cells. In each cell there are infinitely many boxes (enumerated by natural numbers) each containing one real number. In every cell the boxes, enumeration and the real numbers are the same. Now, each mathematician is allowed to open every box but one and then he should guess the real number contained in this only box he hasn't opened. If he doesn't guess the right number, he will be seriously punished. Mathematicians can discuss the strategy before going to prison. Invent a strategy, allowing them to save as many of them as possible.

Pbn
`http://www.math.uni.wroc.pl/~pborod/dydaktyka`