Applications of infinitary combinatorics **11** 2019

Zad. 1 Let (μ_n) be a sequence in the space of probability measures (on a compact space K). Prove that the following conditions are equivalent:

- μ_n is weak* convergent to μ,
- $\limsup_n \mu_n(F) \leq \mu(F)$ for each closed $F \subseteq K$,
- $\liminf_n \mu_n(U) \geq \mu(U)$ for each open $F \subseteq K$,
- $\lim \mu_n(A) = \mu(A)$ for each borel $A \subseteq K$ such that $\mu(\overline{A} \setminus \mathrm{Int}(A)) = 0$.

Zad. 2 Shot that if $\mathcal{A} \subseteq \mathrm{Bor}(K)$ is such that

- $\mathcal{A}$ is closed under finite intersections, and
- every open V is a union of elements of $\mathcal{A}$.

Then, if $\mu_n(A) \to \mu(A)$ for every $A \in \mathcal{A}$, then μ_n converges weak* to μ.

Zad. 3 Use the previous exercises to show that if K is zerodimensional, then μ_n weak* converges to μ if and only if $\mu_n(A) \to \mu(A)$ for every clopen A.

Zad. 4 Prove that if X is $\mathcal{Z}$-Frechet-Urysohn, then it is convexly Frechet-Urysohn.