DST 7 2024

Zad. 1 What is the complexity of the set of ill founded trees in $2^{<\omega}$ as a subset of $\mathbb{T}$, the family of all subtrees in $2^{<\omega}$ (seen as a Polish space in the way described on the lecture)?

Zad. 2 Let $N \subseteq 2^\omega$ be the set of all the sequences with infinitely many 1's. Show that the set

$$\{T \in \mathbb{T} \colon \exists x \in N \ x \in [T]\}$$

is analytic complete.

Zad. 3 Consider the space $X = (\omega \setminus \{0\})^\omega$ and let

$$L = \{x \in X \colon \exists k_0 < k_1 < \dots \ x(k_n) \text{ divides } x(k_{n+1}) \text{ for each } n.\}$$

Show that L is analytic complete. Hint: show that the set of ill founded trees can be reduced to L.

Zad. 4 Show that if E is a Borel (analytic, F_σ, etc.) equivalence relation on X, then $[x]_E$ is Borel (analytic, F_σ, ...) for every x.

Zad. 5 Consider the language $\mathcal{L} = \{<\}$. Show that the set of all the countable $\mathcal{L}$-structures can be seen as a Polish space $X_\mathcal{L} = 2^{\omega\times\omega}$. Show that the isomorphism is an analytic relation on $X_\mathcal{L}$. What if we consider other relational languages?

Zad. 6 Consider the natural action $\mathbb{Z}$ on $\mathbb{R}$ and the induced orbit equivalence relation E (i.e. translations by integers). Show that E is Borel reducible to $Id(\mathbb{R})$.

Zad. 7 Consider the relation E_0 on the Cantor set (i.e. $xE_0y \iff x =^* y$). Show that it is not Borel reducible to $Id([0,1])$). Hint: suppose that $f \colon 2^\omega \to [0,1]$ is such a reduction. Consider an interval $I_n^k = [k/2^n, (k+1)/2^n)$ (where $k < 2^n$) and notice that $f^{-1}[I_n^k]$ is a Borel tail event and so one can apply Kolmogorov 0-1 law.

Zad. 8 Assume Σ^1_1-determinacy. Show that every set $A \in \Sigma^1_1 \setminus \Pi^1_1$ is analytic complete. Hint: use the game which appears in the proof that every set from $\Sigma^0_\alpha \setminus \Pi^0_\alpha$ is Σ^0_α-complete.