ANSWER KEY: SAMPLING DISTRIBUTION OF $\bar{X}$
This answer key provides solutions to the corresponding student activity sheet.

Sampling Distribution of $\bar{X}$

A data set is not provided for these exercises.

Exercise 1

(a) What type of distribution can be used to model dragons' wing spans?

Solution: **D. Bimodal**

(b) What type of distribution can be used to model the average of dragons' wing spans?

Solution: **A. Normal.** The Central Limit Theorem tells us that the sample mean is approximately normally distributed regardless of the shape of the original population distribution.

Exercise 2

Let $X_1, X_2, X_3, \ldots, X_9$ be independent *normal* random variables with mean $\mu_X = 3$ and standard deviation $\sigma_X = 2$. Let $\bar{X}$ be the distribution of the mean of these 9 random variables, namely $\bar{X} = \frac{X_1+X_2+\cdots+X_9}{9}$.

(a) What is the shape of the distribution of $\bar{X}$?

Solution: Since the parent population is normal, then the distribution of $\bar{X}$ is **normal**.

(b) What is the mean of the distribution of $\bar{X}$?

Solution: The same as the mean of the parent population: $\boldsymbol{\mu_{\bar{X}} = 3}$.

(c) What is the standard deviation of the distribution of $\bar{X}$?

Solution: It is the fraction $\frac{1}{\sqrt{9}} = \frac{1}{3}$ of the parent population's standard deviation; $\boldsymbol{\sigma_{\bar{X}} = \frac{2}{\sqrt{9}} = 0.667}$.

(d) Can we determine $P(\bar{X} < 2.5)$ using a z-score? You do not need to compute this probability, just answer **yes** or **no** and briefly explain why or why not.

Solution: Yes. $\bar{X}$ is normally distributed and we know the mean and standard deviation of the parent population. We can convert $x = 2.5$ to a z-score using the mean and standard deviation of $\bar{X}$.

Exercise 3

Let $X_1, X_2, X_3, \ldots, X_{36}$ be independent *skewed* random variables with mean $\mu_X = 3$ and standard deviation $\sigma_X = 2$. Let $\bar{X}$ be the distribution of the mean of these 36 random variables, namely $\bar{X} = \frac{X_1 + X_2 + \cdots + X_{36}}{36}$.

(a) What is the approximate shape of the distribution of $\bar{X}$?

Solution: Most likely, since the sample size is 36, the distribution of $\bar{X}$ is approximately normal. It may depend though on how "skewed" the X_i are. Recall that the sample size required for convergence to normality depends on the shape of the original distribution.

(b) What is the mean of the distribution of $\bar{X}$?

Solution: The same as the mean of the parent population: $\boldsymbol{\mu_{\bar{X}} = 3}$.

(c) What is the standard deviation of the distribution of $\bar{X}$?

Solution: It is the fraction $\frac{1}{\sqrt{36}} = \frac{1}{6}$ of the parent population's standard deviation; $\boldsymbol{\sigma_{\bar{X}} = \frac{2}{\sqrt{36}} = 0.333}$.

(d) Can we determine $P(\bar{X} < 2.5)$ using a z-score? You do not need to compute this probability, just answer **yes** or **no** and briefly explain why or why not.

Solution: Yes, if $\bar{X}$ can be approximated by a normal distribution, which it most likely can be with a sample size of 36. If the X_i are highly skewed, then a larger sample size may be required.

Exercise 4

Let X_1, X_2, X_3, …, X_{100} denote the actual weights of 100 randomly selected bags of sand. The expected weight of each individual bag is μ = 50 pounds and the standard deviation is σ = 1 pound. Let $\bar{X} = \frac{X_1+X_2+\cdots+X_{100}}{100}$.

(a) Assume the bag weights are normally distributed. Randomly select **one** of the 100 bags. What's the probability that it weighs between 49.75 and 50.25 pounds?

Solution: The weight *X* of a single bag has mean μ = 50 pounds and standard deviation σ = 1 pound. We can use a *z*-score or Minitab to determine the desired probability:

$$P(49.75 < X < 50.25) = P\left(\frac{49.75-50}{1} < Z < \frac{50.25-50}{1}\right) = P(-0.25 < Z < 0.25) \cong 0.59871 - 0.40129 = \mathbf{0.19742}$$

(b) Assume the bag weights are normally distributed. What's the probability that the **average weight** $\bar{X}$ of 100 bags is between 49.75 and 50.25 pounds?

Solution: Since the *X*'s are normally distributed, then $\bar{X}$ is normally distributed. The mean and standard deviation of $\bar{X}$ are $\mu_{\bar{X}} = 50$ pounds and $\sigma_{\bar{X}} = \frac{1}{\sqrt{100}} = 0.1$ pound.

$$P(49.75 < X < 50.25) = P\left(\frac{49.75-50}{0.1} < Z < \frac{50.25-50}{0.1}\right) = P(-2.5 < Z < 2.5) \cong 0.99379 - 0.00621 = \mathbf{0.98758}$$

(c) Assume the bag weights are *positively skewed*. Randomly select **one** of the 100 bags. What's the probability that it weighs between 49.75 and 50.25 pounds?

Solution: We cannot determine this since we don't know the distribution of the bag weights.

(d) Assume the bag weights are *positively skewed*. What's the probability that the **average weight** $\bar{X}$ of 100 bags is between 49.75 and 50.25 pounds?

Solution: Since the sample size is 100, then we can assume that $\bar{X}$ is approximately normally distributed and use the same solution from part **(b)**.

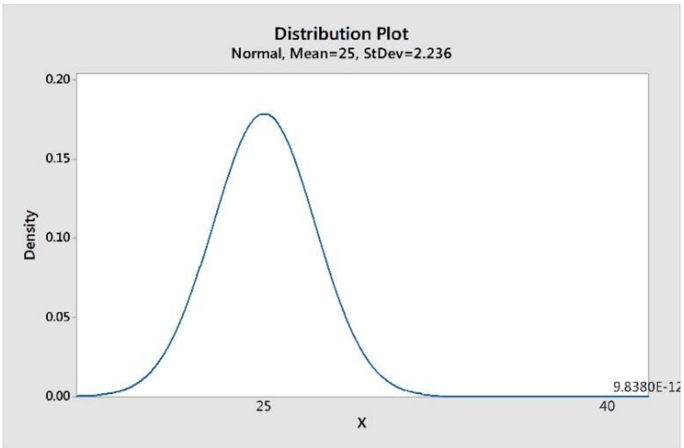
$$P(49.75 < X < 50.25) = P\left(\frac{49.75-50}{0.1} < Z < \frac{50.25-50}{0.1}\right) = P(-2.5 < Z < 2.5) \cong 0.99379 - 0.00621 = \mathbf{0.98758}$$

Exercise 5

A small commuter flight leaves from University Park Airport headed to the O'Hare Airport with 20 passengers. By FAA weight standards for carry-on luggage, a passenger's carry-on luggage should not exceed 40 pounds. Let's assume that the weight of a passenger's carry-on luggage is normally distributed with a mean weight of 25 pounds and a standard deviation of 10 pounds, with only 1 carry-on allowed per passenger. What is the probability that the average luggage weight for the 20 passengers exceeds 40 pounds? Report the z-score.

Solution: Since the luggage weights are normally distributed, then the mean of the luggage weights $\bar{X}$ is also normally distributed with mean $\mu_{\bar{X}} = 25$ pounds and standard deviation $\sigma_{\bar{X}} = 10/\sqrt{20}$ pounds. The probability is approximately 0:

$$P(\bar{X} > 40) = P\left(Z > \frac{40 - 25}{\frac{10}{\sqrt{20}}}\right) \cong P(Z > 6.71) \cong 0$$



Exercise 6

Suppose that Beyonce, Jay Z, and Solange go bowling together. Each of them has a bowling score X that is normally distributed with mean $\mu = 120$ and standard deviation $\sigma = 10$. What is the probability that after bowling one game the average score $\bar{X}$ for the 3 of them is less than 110?

Solution: Since the X's are normally distributed, then so is $\bar{X}$. The mean and standard deviation of the average score $\bar{X}$ are 120 and $10/\sqrt{3} \cong 5.77$, respectively.

$$P(\bar{X} < 110) = P\left(Z < \frac{110 - 120}{\frac{10}{\sqrt{3}}}\right) \cong P(Z < -1.73) \cong \mathbf{0.04182}$$