ANSWER KEY: POPULATION MEAN HYPOTHESIS TESTING FOR LARGE SAMPLES
This answer key provides solutions to the corresponding activity sheet.

Population Mean Hypothesis Testing for Large Samples

The data for these exercises are in the Minitab file ***HypTestForMean_LargeSample_Activity.mtw***.

Exercise 1

For the following multiple choice problems, choose the best answer.

(a) The test statistic and p-value corresponding to the hypothesis test H_0: $\mu = 2$ versus H_1: $\mu > 2$ are $\bar{x} = 2.076$ and $p = 0.065$, respectively. Which of the following is an appropriate interpretation of this p-value?

A. The probability that the null hypothesis is true is 0.065.

B. The probability that the alternative hypothesis is true is 0.065.

C. The data is not normally distributed.

D. In repeated sampling, the probability of observing a test statistic at least as extreme as $\bar{x} = 2.076$ is 0.065.

Solution: Choice A is tempting for most students, but the p-value is not the probability that the null hypothesis is true or false. Similarly, choice B is incorrect. We are not told whether or not the data is from a normal distribution, so there's no support for choice C.

(b) A pudding manufacturer packages its product into bags weighing 1 kilogram, on average. The manufacturer's statistician has discovered that the setting of the machine is causing the fill weights to drift. The statistician needs to detect shifts in the mean weight as quickly as possible and reset the machine when appropriate. In order to detect shifts in the mean weight, he collects a random sample of 50 bags periodically, weighs them, and calculates the mean and standard deviation. The data from this afternoon's sample yields a sample mean of 1.03 kg and a sample standard deviation of 0.08 kg. Determine the *p*-value for the hypothesis test H_0: $\mu = 1$ versus H_1: $\mu \neq 1$.

A. 0.008

B. 0.004

C. 0.011

D. 0.005

E. 0.704

F. 0.996

G. We cannot determine the *p*-value because the data does not come from a normal distribution.

Solution: This is a 1-sample *Z* hypothesis test since the sample size is large, $n = 50$, and the Central Limit Theorem guarantees us that $\bar{X}$ is normally distributed. The null and alternative hypotheses are:

$$H_0: \mu = 1 \text{ versus } H_1: \mu \neq 1.$$

Assuming H_0 is true, the standardized test statistic is:

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1.03 - 1}{\frac{0.08}{\sqrt{50}}} \cong 2.65$$

The test statistic's *p*-value is $2 * P(Z > 2.65) \approx 2 * 0.004 = 0.008$. We multiply $P(Z > 2.65)$ by 2 since the alternative hypothesis is "not equal to."

In Minitab, we obtain:

One-Sample Z

```
Test of μ = 1 vs ≠ 1
The assumed standard deviation = 0.08


 N    Mean  SE Mean        95% CI           Z      P
50  1.0300   0.0113  (1.0078, 1.0522)  2.65  0.008
```

(c) The pain reliever currently used at General Hospital brings relief to patients in a mean time of 3.5 minutes. To compare a new pain reliever with the current one, the new pain reliever is administered to a random sample of $n = 50$ patients. The mean time to feel relief for this sample of patients is 2.8 minutes with a standard deviation of 1.14 minutes.

What are the appropriate null and alternative hypotheses to determine if the mean time for patients to feel relief from the new pain reliever is less than the time required for the hospital's current pain reliever?

A. $H_0 : \mu = 2.8$ versus $H_1: \mu < 2.8$

B. $H_0: \bar{x} = 2.8$ versus $H_1: \bar{x} < 2.8$

C. $H_0 : \mu = 2.8$ versus $H_1: \mu > 2.8$

D. $H_0 : \mu = 3.5$ versus $H_1: \mu < 3.5$

E. $H_0 : \bar{x} = 3.5$ versus $H_1: \bar{x} < 3.5$

F. $H_0 : \mu = 3.5$ versus $H_1: \mu > 3.5$

Solution: Choices B and E are incorrect since hypothesis tests are performed on population parameters, and not sample statistics. We are trying to determine if the new pain reliever requires "less time" to give the patient relief, and so we want a "less than" alternative hypothesis. This leaves us with choices A and D. We are determining if the new pain reliever brings patients relief in less time than the population mean time for the current pain reliever, which is 3.5 minutes.

(d) We are testing $H_0: \mu = 2$ versus $H_1: \mu > 2$ and conclude that we can reject the null hypothesis H_0 at significance level $\alpha = 0.05$. Suppose we decide to change the alternative hypothesis from $H_1: \mu > 2$ to $H_1: \mu \neq 2$. Using the same data, can we still reject H_0 at $\alpha = 0.05$?

A. Yes

B. No

C. There is not enough information to answer this question.

Solution: This problem tests a student's ability to correctly assess the relationship between a *p*-value and level of significance α.

Since $H_0: \mu = 2$ versus $H_1: \mu > 2$ is rejected at $\alpha = 0.05$, then the *p*-value for this hypothesis test must be less than 0.05. Since the alternative hypothesis is "greater than," then the *p*-value is calculated in the right tail of the distribution of $\bar{X}$. If the alternative hypothesis is changed to $H_1: \mu \neq 2$, then the *p*-value needs to be doubled. Since all we know is that $p < 0.05$ for a one-tailed test, then the *p*-value is less than 0.1 for a two-tailed test. Depending on the original one-tailed *p*-value, we may or may not end up rejecting H_0. For example, if the original *p*-value is less than 0.025, then when it's doubled, it's still less than 0.05. On the other hand, if the original *p*-value is between 0.025 and 0.05, then when it's doubled, it's greater than 0.05.

(e) We are conducting the following hypothesis test: H_0: $\mu = 9.5$ versus H_1: $\mu > 9.5$. The approximate *p*-value for this hypothesis test is:

A. 0.015

B. 0.031

C. 0.058

D. 0.267

E. 0.395

F. 0.971

G. We cannot determine the *p*-value because we don't know if the data is from a normal distribution.

Solution: This is a 1-sample *Z* hypothesis test since the sample size is large, $n = 50$, and the Central Limit Theorem guarantees us that $\bar{X}$ is normally distributed. The null and alternative hypotheses are:

$$H_0: \mu = 9.5 \text{ versus } H_1: \mu > 9.5$$

Assuming H_0 is true, the standardized test statistic is:

$$z = \frac{9.728 - 9.5}{\frac{0.86}{\sqrt{50}}} \cong \frac{9.728 - 9.5}{0.122} \cong 1.87$$

The test statistic's *p*-value is $P(Z > 1.87) \approx$ **0.031**.

In Minitab, we obtain:

One-Sample Z: Drying Time (hours)

```
Test of μ = 9.5 vs > 9.5
The assumed standard deviation = 0.86

Variable              N   Mean  StDev  SE Mean  95% Lower Bound     Z      P
Drying Time (hours)  50  9.728  0.958    0.122            9.528  1.87  0.031
```

(f) A certain brand of orange juice is advertised to contain 85% fruit juice per bottle. A random sample of 32 bottles of this juice is selected in order to perform the hypothesis test:

$$H_0: \mu = 0.85 \quad \text{versus} \quad H_1: \_\_\_\_\_,$$

where μ represents the average percentage of juice per bottle.

What is the correct alternative hypothesis for this test?

A. $H_1: \mu < 0.82$

B. $H_1: \mu < 0.85$

C. $H_1: \mu > 0.82$

D. $H_1: \mu > 0.85$

E. $H_1: \mu \neq 0.82$

F. $H_1: \mu \neq 0.85$

Solution: This is a 1-sample *Z* hypothesis test since the sample size is large, $n = 32$, and the Central Limit Theorem guarantees us that $\bar{X}$ is normally distributed. Choices A, C, and E are incorrect since hypothesis tests are performed on population parameters, and not sample statistics. Since the *z*-score is in the left tail of a standard normal distribution and the *p*-value is less than 0.5, then either the alternative hypothesis is "less than" or "not equal to."

Using Minitab, $P(Z < -0.81) \approx 0.2090$. Since the reported value for *p* is 0.42 in the Minitab output, then we can see that $P(Z < -0.81)$ has been doubled. Thus, the alternative hypothesis is $H_1: \mu \neq 0.85$.

(g) A quality control specialist takes several measurements to test $H_0: \mu = 2$ versus $H_1: \mu \neq 2$. She computes a *p*-value of 0.01 for the hypothesis test. Which interpretation is correct?

A. The probability that $\mu = 2$ is 0.01

B. The probability that $\mu \neq 2$ is 0.01.

C. The probability that the quality control specialist conducted the study properly is 0.99.

D. The probability of the quality control specialist observing those results (or more extreme) if $\mu = 2$ is 0.01.

Solution: As in **Exercise 1**, part **(a)**, choice A is tempting for most students, but the *p*-value is not the probability that the null hypothesis is true or false. Similarly, choice B is incorrect. We don't know if the quality control specialist conducted the study properly – there is no way of knowing this from the information given. The definition of the *p*-value is provided as choice D.

(h) Which of the following statements is <u>not</u> true regarding hypothesis testing?

A. The alternative hypothesis is the assertion that is contradictory to the null hypothesis.

B. The null hypothesis is the claim that is assumed to be true, or the “status quo” hypothesis.

C. The sample evidence is used to determine whether or not to reject the alternative hypothesis.

D. The two possible conclusions from a hypothesis test are reject H_0 or fail to reject H_0.

E. When drawing a conclusion after performing a hypothesis test, there is always a chance that you made the wrong decision, even if that chance is very small.

Solution: Choice C is incorrect since we make decisions in a hypothesis test with respect to the null hypothesis.

(i) In performing the hypothesis test H_0: $\mu = 8$ versus H_1: $\mu \neq 8$, the resulting *p*-value is 0.016. Thus, if we construct a 95% confidence interval for μ using the exact same data, $\mu = 8$ will not be included in the 95% confidence interval.

A. True

B. False

C. We do not have enough information to determine this.

Solution: Since the *p*-value is less than 0.05, then $\mu = 8$ will not be included in a 95% confidence interval for the population mean.

(j) You perform the hypothesis test H_0: $\mu = 180$ versus H_1: $\mu > 180$ for the average number of minutes per day that students at your school watch TV. Which of the following statements is correct regarding the *p*-value?

A. Assuming the null hypothesis is true, an extremely small *p*-value indicates that the sample mean calculated from the sample data is extremely different from null mean μ = 180.

B. The *p*-value measures the probability that the alternative hypothesis is true.

C. The *p*-value measures the probability that the null hypothesis is true.

D. The larger the *p*-value, the stronger the evidence against the null hypothesis.

E. A large *p*-value indicates that the data supports the alternative hypothesis.

Solution: As in **Exercise 1**, parts **(a)** and **(g)**, choices B and C are incorrect. Choices D and E are incorrect since a larger *p*-value supports not rejecting the null hypothesis. The correct choice is A.

For example, if we had a value "extremely different" from 3, then its z-score will be large and in the right tail of the distribution of $\bar{X}$. The larger the z-score in the right tail, the smaller the p-value.

(k) Under normal environmental conditions, adult catfish in Dog Lake have an average length of μ = 13.9 cm with a standard deviation σ = 2.1 cm. Students who frequently fish at Dog Lake claim that the catfish are smaller than usual this year. Suppose your statistics class takes a random sample of adult catfish from Dog Lake. Which of the following provides the **strongest** evidence to support the claim that students are catching smaller than average length (13.9 cm) catfish this year?

A. A random sample of size n = 36 with a sample mean of $\bar{x} = 13.5$ cm.

B. A random sample of size n = 36 with a sample mean of $\bar{x} = 13.3$ cm.

C. A random sample of size *n* = 121 with a sample mean of $\bar{x} = 13.5$ cm.

D. A random sample of size n = 121 with a sample mean of $\bar{x} = 14.5$ inches.

E. We do not have enough information to determine this.

Solution: This is a 1-sample Z hypothesis test since both sample sizes, n = 36 and n = 121, are larger than 30. The null and alternative hypotheses are:

$$H_0: \mu = 13.9 \text{ versus } H_1: \mu < 13.9$$

For a "less than" alternative hypothesis, the strongest evidence against H_0 will be the sample that has the largest ***negative*** z-score (i.e. the z value in the left tail of the standard normal distribution that is furthest from mean μ = 0). Using Minitab, z-score's for choices A through D are:

A: One-Sample Z

```
Test of μ = 13.9 vs < 13.9
The assumed standard deviation = 2.1

 N    Mean  SE Mean  95% Upper Bound      Z      P
36  13.500    0.350           14.076  -1.14  0.127
```

B: One-Sample Z

```
Test of μ = 13.9 vs < 13.9
The assumed standard deviation = 2.1

 N    Mean  SE Mean  95% Upper Bound      Z      P
36  13.300    0.350           13.876  -1.71  0.043
```

C: One-Sample Z

```
Test of μ = 13.9 vs < 13.9
The assumed standard deviation = 2.1

  N    Mean  SE Mean  95% Upper Bound      Z      P
121  13.500    0.191           13.814  -2.10  0.018
```

D: One-Sample Z

```
Test of μ = 13.9 vs < 13.9
The assumed standard deviation = 2.1

  N    Mean  SE Mean  95% Upper Bound     Z      P
121  14.500    0.191           14.814  3.14  0.999
```

(l) You just bought a new laptop with a supposed true population mean battery life of 3 hours. You survey friends who have the same laptop to determine if, in fact, the true population mean is less than 3 hours. For a random sample of $n = 40$ friends, you calculate a sample mean of 3.25 hours. To test the “less than” speculation, you should use the following hypothesis:

$$H_0: \mu = 3 \quad \text{versus} \quad H_1: \mu < 3$$

A. True

B. False

C. We do not have enough information to determine this.

(m) The *Central Limit Theorem*, the key to performing a hypothesis test for the population mean μ with a *Z* test, states that:

A. We can always use a normal curve to approximate the distribution of the sample mean $\bar{X}$.

B. If *n* is large (e.g. $n > 30$) and the original population distribution is normal, then the distribution of the sample mean $\bar{X}$ can be approximated by a normal curve.

C. We can always use a normal curve to approximate the distribution of a random variable *X*.

D. If n is large (e.g. n > 30) then the distribution of the sample mean $\bar{X}$ can be approximated closely by a normal curve even if the original distribution is not normal.

E. If n is large (e.g. n > 30), then the distribution of the random variable X can be approximated closely by a normal curve even if the original distribution is not normal.

Solution: The Central Limit Theorem is stated in choice D.

Exercise 2

Putting Puzzles Together

The answers below are for the data provided in the "Puzzle Times (secs)" column. Answers will change if class data is used instead.

(a) Can we assume that the distribution of the mean time $\bar{X}$ for n = 43 employees is normally distributed? Why or why not? Be specific.

Solution: **Yes!** Since there are n = 43 data points, then the Central Limit Theorem says that the distribution of the mean times $\bar{X}$ is normally distributed.

(b) Determine the standardized test statistic z and its p-value by hand.

Solution:

$$z = \frac{95.65 - 90}{28.09/\sqrt{43}} \cong \mathbf{1.32}$$

The p-value is $P(Z > 1.32) \approx$ **0.094**.

(c) Determine the standardized test statistic z and its p-value in Minitab.

Solution:

One-Sample Z: Puzzle Times (secs)

```
Test of μ = 90 vs > 90
The assumed standard deviation = 28.09


Variable              N   Mean  StDev  SE Mean  95% Lower Bound     Z      P
Puzzle Times (secs)  43  95.65  28.09     4.28            88.61  1.32  0.094
```

(d) Can we reject H_0 at an α = 0.10 level of significance? Why or why not?

Solution: Since the p-value 0.094 is less than $\alpha = 0.10$, then we **reject H_0**. There is sufficient evidence at the $\alpha = 0.10$ significance level to suggest that the puzzle takes more than 90 seconds to assemble.

(e) Can we reject H_0 at an $\alpha = 0.05$ level of significance? Why or why not?

Solution: Since the p-value 0.094 is greater than $\alpha = 0.05$, then we **do not reject H_0**. There is not sufficient evidence at the $\alpha = 0.05$ significance level to suggest that the puzzle takes more than 90 seconds to assemble.

Exercise 3

Cutting Paper Strips

The answers below are for the data provided in the "Cut Lengths (cm)" column, using μ_0 = 14 cm. Answers will change if class data is used instead.

Perform the following three hypothesis tests in Minitab, estimating σ by calculating the standard deviation of the data. What conclusions can you draw from the p-values reported for each hypothesis test?

(a) $H_0: \mu = \mu_0$ versus $H_1: \mu \neq \mu_0$

Solution:

Descriptive Statistics: Cut Lengths (cm)

Variable	Total Count	Mean	SE Mean	StDev	Median
Cut Lengths (cm)	25	12.434	0.242	1.212	12.750

One-Sample Z: Cut Lengths (cm)

```
Test of μ = 14 vs ≠ 14
The assumed standard deviation = 1.212
```

Variable	N	Mean	StDev	SE Mean	95% CI	Z	P
Cut Lengths (cm)	25	12.434	1.212	0.242	(11.959, 12.909)	-6.46	0.000

The standardized test statistic and p-value are -6.46 and 0, respectively. At $\alpha = 0.05$, we reject H_0 since the p-value is 0 and conclude the data suggests that the true mean is not 14 cm.

(b) $H_0: \mu = \mu_0$ versus $H_1: \mu < \mu_0$

Solution:

One-Sample Z: Cut Lengths (cm)

```
Test of μ = 14 vs < 14
The assumed standard deviation = 1.212


Variable            N    Mean  StDev  SE Mean  95% Upper Bound      Z      P
Cut Lengths (cm)   25  12.434  1.212    0.242           12.833  -6.46  0.000
```

The standardized test statistic and *p*-value are the same as in part **(a).** The appropriate decision is to reject H_0. Clearly, the evidence suggests that the true mean μ is not 14, but less.

(c) $H_0: \mu = \mu_0$ versus $H_1: \mu > \mu_0$

Solution:

One-Sample Z: Cut Lengths (cm)

```
Test of μ = 14 vs > 14
The assumed standard deviation = 1.212


Variable            N    Mean  StDev  SE Mean  95% Lower Bound      Z      P
Cut Lengths (cm)   25  12.434  1.212    0.242           12.035  -6.46  1.000
```

The test statistic is still -6.46, but since the alternative hypothesis is "greater than," then the area or probability to the right of z = -6.46 on a standard normal curve is 1. With a p-value of 1, we do not reject H_0.

Exercise 4

A drug manufacturer claims a given type of medicine contains 2.5 milligrams of a certain active ingredient per capsule. An independent laboratory takes a random sample of 20 of these capsules and measures the amount of the active ingredient in each, in order to determine if the true mean amount of the active ingredient is actually less than 2.5.

(a) Using the proper statistical notation, write down the null and alternative hypotheses.

Solution:

$$H_0: \mu = 2.5 \quad \text{versus } H_1: \mu < 2.5$$

(b) What does the parameter of interest μ represent in words? Select the best answer.

A. The true mean amount of the ingredient in all capsules of this type.

B. The true mean amount of the ingredient in the 20 capsules of the laboratory's sample.

C. The difference between the true mean amount of the ingredient in the population of all capsules and the sample mean amount of the ingredient in the 20 capsules in our sample.

D. The true proportion of all capsules of this type that contain less than 2.5 milligrams of the ingredient.

E. The amount of the ingredient in a randomly selected capsule of this type.

(c) Using Minitab, determine the mean and standard deviation of the sample data provided. The data is in the Minitab column "Active Ingredient."

Solution:

Descriptive Statistics: Active Ingredient

Variable	Total Count	Mean	SE Mean	StDev	Minimum	Median	Maximum
Active Ingredient	20	2.004	0.176	0.787	0.610	1.995	3.310

(d) Assume that the amount of the ingredient in capsules is normally distributed. Also, assume that the laboratory is told that the population standard deviation is σ = 0.8 mg. By hand, calculate the z statistic and *p*-value for the hypothesis test in part **(a)** based on the sample data provided.

Solution: Since the amount of the ingredient in capsules is normally distributed, then the distribution of the sample mean $\bar{X}$ is also normally distributed. Assuming H_0 is true, the standardized test statistic is:

$$z = \frac{2.004 - 2.5}{\frac{0.8}{\sqrt{20}}} \cong \frac{2.004 - 2.5}{0.176} \cong \mathbf{-2.77}$$

The test statistic's *p*-value is $P(Z <) \approx$ **0.003**.

(e) Verify the z statistic and *p*-value using Minitab. From the 1-Sample Z dialog box, click **Options** to select the appropriate alternative hypothesis.

Solution:

One-Sample Z: Active Ingredient

```
Test of μ = 2.5 vs < 2.5
The assumed standard deviation = 0.8


Variable             N   Mean  StDev  SE Mean  95% Upper Bound      Z      P
Active Ingredient   20  2.004  0.787    0.179            2.298  -2.77  0.003
```

Exercise 5

A waiter randomly samples times for n = 40 tables on busy Saturday nights and obtains a sample mean of $\bar{x} = 4.2$ minutes with a sample standard deviation of s = 0.8 minutes. The sample data is used to construct the following 95% confidence interval for the true mean clean-up time μ: [3.952, 4.448].

Based on the 95% confidence interval, can the null hypothesis for the following hypothesis test be rejected at an α = 0.05 level of significance? Explain why or why not.

H_0: μ = 3.5 minutes   versus H_1: μ ≠ 3.5 minutes

Solution: Yes, the null hypothesis should be rejected at α = 0.05. The "easiest" way to answer this question is to see that 3.5 is not contained in the 95% confidence interval for the population mean μ. Thus, the *p*-value for H_0: μ = 3.5 versus H_1: μ ≠ 3.5 is less than 0.05. At an α = 0.05 level of significance, we would reject H_0: μ = 3.5.

Using Minitab, we can confirm that we should reject H_0 since the *p*-value is approximately 0.

One-Sample Z

```
Test of μ = 3.5 vs ≠ 3.5
The assumed standard deviation = 0.8


 N   Mean  SE Mean       95% CI          Z      P
40  4.200    0.126  (3.952, 4.448)  5.53  0.000
```

Exercise 6

A random sample of n = 50 drill bits is used to put holes into a steel doorframe. The lifetime of a drill bit is measured as the number of holes drilled before the bit fails. The average lifetime of a drill bit is 12.68 holes with a standard deviation of 6.83 holes. Calculate the z statistic and *p*-value for the following hypothesis tests by hand or in Minitab.

(a) H_0: μ = 12 holes versus H_1: μ > 12 holes

(b) H_0: μ = 12 holes versus H_1: μ ≠ 12 holes

(c) H_0: μ = 12 holes versus H_1: μ < 12 holes

Solutions:

One-Sample Z

```
Test of μ = 12 vs > 12
The assumed standard deviation = 4.83

 N    Mean  SE Mean  95% Lower Bound     Z      P
50  12.680    0.683           11.556  1.00  0.160
```

One-Sample Z

```
Test of μ = 12 vs ≠ 12
The assumed standard deviation = 4.83

 N    Mean  SE Mean       95% CI            Z      P
50  12.680    0.683  (11.341, 14.019)  1.00  0.319
```

One-Sample Z

```
Test of μ = 12 vs < 12
The assumed standard deviation = 4.83

 N    Mean  SE Mean  95% Upper Bound     Z      P
50  12.680    0.683           13.804  1.00  0.840
```

Exercise 7

According to a Google search, the average height of male soccer players in the U.S. is normally distributed with mean 1.79 m with a standard deviation of 0.04 m.

(a) For a randomly selected soccer team of 11 players, what is the probability that the *average* height of the players is less than 1.77 m? Calculate the probability by hand or in Minitab.

Solution: Let *X* represent the heights of U.S. male soccer players. Then *X* has a normal distribution with mean μ = 1.79 m and standard deviation σ = 0.04 m. Since *X* is normally distributed, then $\bar{X}$ is normally distributed. The mean of the distribution of $\bar{X}$ is $\mu_{\bar{X}} = 1.79$ and the standard deviation of $\bar{X}$ is $\sigma_{\bar{X}} = \frac{0.04}{\sqrt{11}} \cong 0.012$.

Determining the probability by hand, we obtain:

$$P(\bar{X} < 1.77) \cong P\left(Z < \frac{1.77 - 1.79}{0.012}\right) \cong P(Z < -1.67) \cong \mathbf{0.0475}$$

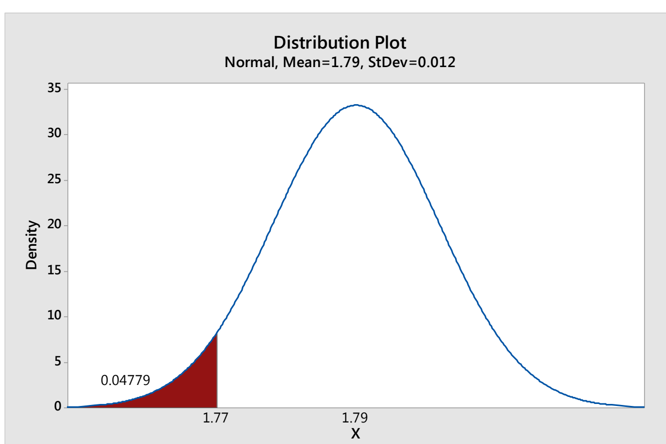
Recall that in Minitab there's a very easy way to determine this probability.

Minitab 17

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *1.79*.
5 In **Standard Deviation**, type *0.012*.
6 Click the **Shaded Area** tab. Under **Define Shaded Area By**, choose **X Value**.
7 Click **Left Tail**, since we want the probability that a player's height *is less than* 1.77. In **X value**, type *1.77*.
8 Click **OK**.

Minitab Express

1 Open the display probability dialog box.
   - Mac: **Statistics** > **Probability Distributions** > **Distribution Plots > Display Probability**
   - PC: **STATISTICS** > **Distribution Plot > Display Probability**
2 From **Distribution**, select **Normal**.
3 In **Mean**, enter *1.79*.
4 In **Standard Deviation**, enter *0.012*.
5 Under **Shade the area corresponding to the following**, select **A specified x value**.
6 Click the **Left Tail** icon, since we want the probability that a player's height *is less than* 1.77.
7 In **X value**, enter *1.77*.
8 Click **OK**.



(b) In view of the small sample size, must you make any additional assumptions to justify the answer to part **(a)**? Please provide a short explanation.

Solution: No. The distribution of the heights is normally distributed, and so the distribution of the mean heights $\bar{X}$ is also normally distributed.

(c) Set up the hypothesis test to test the true average height of male U.S. soccer players.

Solution:

$$H_0: \mu = 1.79 \quad \text{versus} \quad H_1: \mu < 1.79$$

(d) Determine the z statistic and *p*-value for the hypothesis test in part **(c)**.

Solution: We already have these values from part **(a)**, namely the standardized test statistic is $z = -1.67$ and the *p*-value is 0.0475 (table) or 0.0478 (Minitab – more accurate).

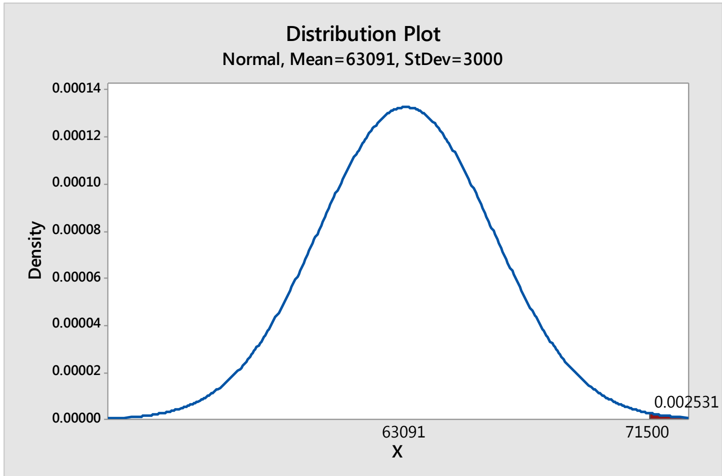
Exercise 8

According to a study, the U.S. mean family income is $63,091 with a standard deviation of $21,000.

(a) If a consulting agency surveys 49 families at random, what is the probability that it finds a *mean* family income that is more than $71,500? Calculate the probability by hand or in Minitab.

Solution: Let X represent family incomes in the U.S. We don't know the distribution of X, but we know its mean is $\mu = 63091$ and its standard deviation is $\sigma = 21000$. The distribution of the mean family income is $\bar{X}$. Since $\bar{X}$ is the distribution of the mean of $n = 49$ family incomes (i.e. n "large"), then $\bar{X}$ will be approximately normally distributed. The mean of $\bar{X}$ is $\mu_{\bar{X}} = 63091$ and the standard deviation of $\bar{X}$ is $\sigma_{\bar{X}} = \frac{21000}{\sqrt{49}} = 3000$.

Using Minitab to evaluate, we obtain the *p*-value **0.0025**.



(b) For the hypothesis test H_0: $\mu = 63{,}091$ versus H_1: $\mu \neq 63{,}091$, determine the z statistic and *p*-value.

Solution: The standardized test statistic is:

$$z = \frac{71500 - 63091}{3000} = \mathbf{2.803}$$

From part **(a)**, the *p*-value is **0.0025**.

(c) Suppose the median income in the U.S. was $55,000. Why do many websites use the median as their indicator of income level instead of the mean?

Solution: Because several very rich people (e.g. doctors, lawyers) in a city can “skew” the distribution so that the average of the distribution is quite a bit larger than the median value. Incomes, in general, are skewed distributions. In this case, the medians are better representatives as the “true middle” of the distribution.

Exercise 9

Your neighbor grows and sells cucumbers in the summer. She packages them in plastic storage bags and claims that the true mean weight of one of these bags is 1 pound. To test her claim, you take a random sample of 64 of these cucumber-filled bags, weigh them, and record their weights in the Minitab column “Bag Weights (lbs).”

With 90% confidence, is her claim “the mean weight of these bags is 1 pound” true? Use either a hypothesis test or confidence interval to provide supporting evidence for or against this claim.

Solution: Using Minitab, the 90% confidence interval for μ is:

One-Sample Z: Bag Weights (lbs)

The assumed standard deviation = 0.5042

Variable	N	Mean	StDev	SE Mean	90% CI
Bag Weights (lbs)	64	0.8771	0.5042	0.0630	(0.7734, 0.9807)

The 90% confidence interval for μ does not contain 1 pound, so her claim is not supported with 90% confidence.

The appropriate hypothesis test to conduct is: H_0: μ = 1 versus H_1: μ ≠ 1 at significance level α = 0.10. The Minitab output is:

One-Sample Z: Bag Weights (lbs)

Test of μ = 1 vs ≠ 1
The assumed standard deviation = 0.5042

Variable	N	Mean	StDev	SE Mean	90% CI	Z	P
Bag Weights (lbs)	64	0.8771	0.5042	0.0630	(0.7734, 0.9807)	-1.95	0.051

According to the *p*-value 0.051, we can reject H_0 at the α = 0.10 level of significance. Thus, the claim is not supported with 90% confidence.

It's interesting to note that if α were 0.05, then we are in the "grey area" of rejecting or not rejecting H_0. The 95% confidence interval just "barely" captures μ = 1. According to the *p*-value, we would just "barely" want to not reject H_0 at α = 0.05.

One-Sample Z: Bag Weights (lbs)

Test of μ = 1 vs ≠ 1
The assumed standard deviation = 0.5042

Variable	N	Mean	StDev	SE Mean	95% CI	Z	P
Bag Weights (lbs)	64	0.8771	0.5042	0.0630	(0.7536, 1.0006)	-1.95	0.051