Minitab

ACTIVITY SHEET: POPULATION MEAN CONFIDENCE INTERVALS FOR LARGE SAMPLES

This activity sheet includes exercises to assess students' understanding of important concepts presented in the *Population Mean Confidence Intervals for Large Samples* lesson.

Population Mean Confidence Intervals for Large Samples

The data for these exercises are in the Minitab file ***CIForMean__LargeSample_Activity.mtw***.

Exercise 1

Suppose that n = 100 of your favorite brand of candy bar are randomly sampled and their weights (in grams) are measured. A 95% confidence interval for the mean weight is $42 \le \mu \le 48$. Choose the best answer for each of these.

(a) Would a 99% confidence interval calculated from the same sample data be tighter or wider?

Tighter or Wider

(b) Consider the following statement: There is a 95% chance that μ is between 42 and 48.
Note: If you answer true, then you are saying that μ is a random entity.

True or False

(c) Consider the following statement: There is a 95% chance that the interval [42, 48] contains μ.
Note: If it is true, then you are saying that the confidence interval is a random entity.

True or False

(d) Consider the following statement: If n = 100 candy bars are randomly sampled and the 95% confidence interval for μ was computed, and this process were repeated 1000 times, then approximately 950 of the confidence intervals would contain the true value of μ.

True or False

Exercise 2

The *Central Limit Theorem*, the key to building a confidence interval for the population mean μ with a Z test, states which one of the following:

A. We can always use a normal curve to approximate the distribution of the sample mean $\bar{X}$.

B. If n is large (e.g. $n > 30$) and the original population is normal, then the distribution of the sample mean $\bar{X}$ can be approximated by a normal curve.

C. We can always use a normal curve to approximate the distribution of sample values X.

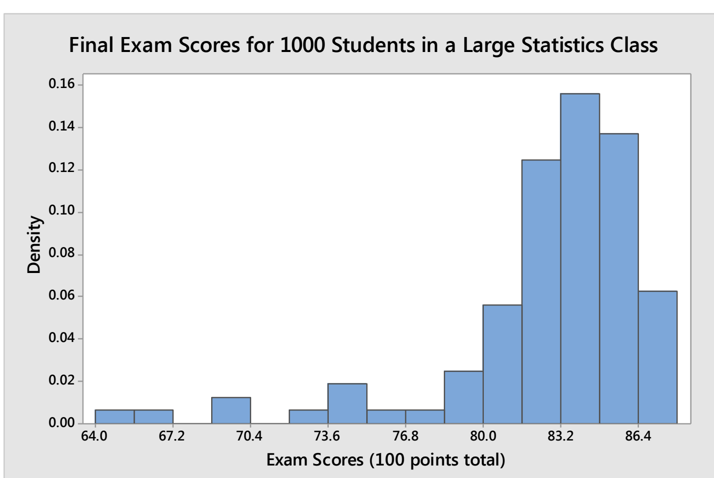
D. If n is large (e.g. $n > 30$) then the distribution of the sample mean $\bar{X}$ can be approximated by a normal curve even if the original distribution is not normal.

E. If n is large (e.g. $n > 30$), then the distribution of the sample values X can be approximated by a normal curve even if the original distribution is not normal.

Exercise 3

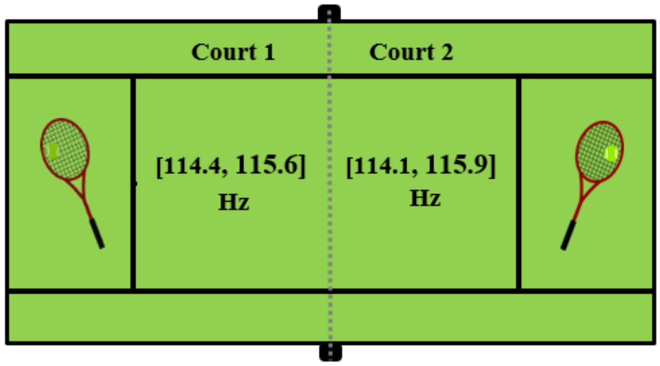
You choose 36 scores from the population of all students' final exam scores (below) and calculate the mean $\bar{x}$ of those test scores. You repeat this process 100 times and plot the distribution $\bar{X}$ of the 100 sample means, the $\bar{x}$'s. The distribution $\bar{X}$ of the sample means will be approximately normal.

True or False



Exercise 4

Each tennis court contains a confidence interval for the true population mean resonance frequency (Hz) for all tennis rackets of a certain type. Both confidence intervals use the same sample mean, sample standard deviation, and sample size; the only difference between them is the confidence level.



(a) What is the value of the sample mean resonance frequency $\bar{x}$ used to construct both confidence intervals?

(b) Both intervals were calculated from the same sample data, but the confidence level for one of these intervals is 90% and the other is 99%. Which interval, [114.4, 115.6] or [114.1, 115.9], has a confidence level of 90%? Why?

Exercise 5

The “House of Blues” in Boston is planning its budget for next year. In estimating the security man-hours needed for concerts, the average length of concerts is required. Using a random sample of n = 36 previous concerts, the sample mean duration is $\bar{x}$ = 110 minutes, while the population standard deviation is σ = 20 minutes. Determine the 95% confidence interval for the true mean duration μ of concerts using the sample data.

(a) Construct the 95% confidence interval by hand.

(b) Construct the 95% confidence interval in Minitab:

1 Open the 1-sample Z dialog box.
- Minitab 17: **Stat > Basic Statistics > 1-Sample Z**
- Minitab Express Mac: **Statistics > 1-Sample Inference > Z**
- Minitab Express PC: **STATISTICS > One Sample > Z**

2 From the drop-down list, select **Summarized data**.
3 In **Sample size**, enter *36*.
4 In **Sample mean**, enter *110*.
5 In **Known standard deviation**, enter *20*.
6 Click **OK**.

Exercise 6

A random sample of n = 50 drill bits is used to put holes into a steel doorframe. The lifetime of a drill bit is measured as the number of holes drilled before the bit fails. The average lifetime of a drill bit is 12.68 holes with a standard deviation of 6.83 holes. Which formula should be used to determine a 60% confidence interval for the mean lifetime of the drill bits?

A. 12.68 ± 0.25 * (6.83/√50)	B. 12.68 ± 0.25 * (6.83/50)	C. 12.68 ± 0.25 * (6.83^2/50)
D. 12.68 ± 0.84 * (6.83/√50)	E. 12.68 ± 0.84 * (6.83/50)	F. 12.68 ± 0.84 * (6.83^2/50)
G. 12.68 ± 1.28 * (6.83/√50)	H. 12.68 ± 1.28 * (6.83^2/√50)	I. 12.68 ± 1.28 * (6.83*√50)

Exercise 7

A neighbor grows and sells cucumbers in the summer. She packages them in plastic storage bags and claims that the true mean weight of these bags is 1 pound.

To test her claim, you take a random sample of 64 of these cucumber-filled bags, weigh them, and find the sample mean weight to be $\bar{x}$ = 1.03 pounds with a sample standard deviation of s = 0.08 lbs.

(a) Determine a 90% confidence interval for the population mean weight μ of these bags of cucumbers by hand. Then check your results using Minitab.

Minitab 17

1 Choose **Stat > Basic Statistics > 1-Sample Z**.
2 Choose **Summarized data**.
3 In **Sample size**, enter *64*.
4 In **Sample mean**, enter *1.03*.
5 In **Known standard deviation**, enter *008*.
6 Click **Options**. In **Confidence level**, enter *90*. Click **OK** in each dialog box.

Minitab Express

1 Open the 1-sample Z dialog box.
    - Mac: **Statistics** > **1-Sample Inference** > **Z**
    - PC: **STATISTICS** > **One Sample** > **Z**
2 From the drop-down list, select **Summarized data**.
3 In **Sample size**, enter *64*.
4 In **Sample mean**, enter *1.03*.
5 In **Known standard deviation**, enter *008*.
6 Click the **Options** tab. Under **Confidence level**, select *90*.
7 Click **OK**.

(b) With 90% confidence, is the neighbor’s claim true? Why or why not?

Exercise 8

For a confidence interval, as the confidence level increases, the reliability goes ____________ and the precision goes _______________.

A. up ... up   B. down ... up   C. down ... down   D. up ... down

Exercise 9

(a) A large box contains 10,000 ball bearings. A random sample of 120 bearings is chosen. The sample mean diameter is $\bar{x}$ = 10 mm, and the standard deviation is s = 0.24 mm. A 95% confidence interval for the true mean diameter of the 10,000 bearings is 10 ± 1.96 * 0.24 / $\sqrt{10000}$.

True or False

(b) For large n (i.e. n > 30), a two-sided 95% confidence interval for the population mean μ will contain the sample mean $\bar{x}$ approximately 95% of the time.

True or False

(c) Suppose that a random sample of n = 50 bottles of a certain brand of cough syrup is selected, and a 95% confidence interval for the true mean alcohol content in these bottles is calculated as [7.8, 9.4] mg. A 99% confidence interval calculated from the same sample data would be wider.

True or False

Exercise 10

Moms' Ages

If students are helping to conduct this exercise in class, at least 30 students are needed to be able to apply the concepts, namely a 1-sample *z* confidence interval for the population mean. Instructions for creating the data set are below.* If you do not have time to collect class data or do not have at least 30 students to perform this activity, then you can use the 'Moms Ages (yrs)' data found in ***CIForMean__LargeSample_Activity.mtw***.

* At the beginning of class, ask students to go to the board as they are working on these exercises to record their Moms' ages. Alternatively, students can record their Moms' ages anonymously on a sheet of paper to submit. Once everyone has recorded his or her Mom's age, the instructor can enter the data in Minitab. The instructor can then email the worksheet data to students to ensure that everyone is working with the same data set.

(a) In Minitab, determine the sample standard deviation s for Moms' ages. If you don't remember how to do this, see the ***Describing Data Numerically*** lesson.

(b) Using your class as a random sample of students who are "similar" to the large student population (e.g. student ages, similar majors, etc.), determine a two-sided 90% confidence interval for the true mean age μ of Moms. Construct this interval by hand or in Minitab. Recall that you'll need to use the sample standard deviation s in place of the unknown population standard deviation σ.

(c) Does the above confidence interval actually contain the true mean age μ for Moms' ages for students in this category?

A. Yes B. No C. We can't be sure

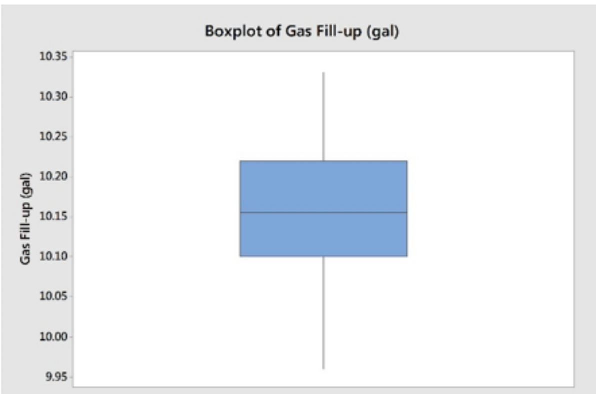
(d) Why can we assume that $\bar{X}$ is approximately normally distributed?

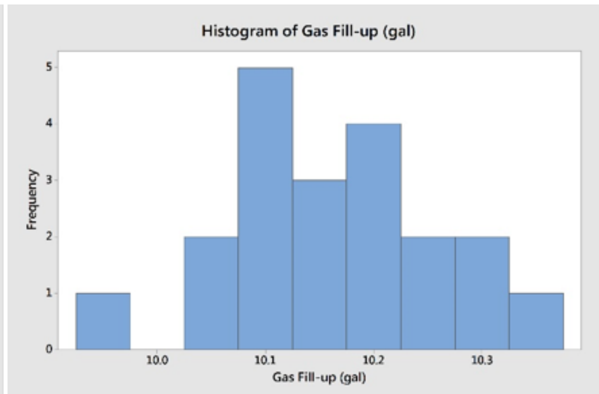
Exercise 11

Your Saturn 3-door was a great car. The owner's manual said that it had a 10-gallon gas tank. You kept track of the number of gallons of gas needed to fill up the tank once the "Low Fuel" icon lit up on the dashboard. Here are the results of 20 random fill-ups (in gallons):

10.27	10.15	10.18	10.05	10.08	10.11	10.12	10.29	10.10	10.19
9.96	10.14	10.16	10.29	10.23	10.19	10.10	10.33	10.06	10.18

Here are graphical views of the data, as a boxplot and a histogram:





(a) Let *X* represent the number of gallons of gas needed to fill your car's gas tank. In order to use the formula from the lesson to construct a confidence interval for the true mean number of gallons of gas μ to fill your gas tank, we have to assume that $\bar{X}$ is normally distributed. Why is this a reasonable assumption based on the boxplot and histogram? Please note that this question is asking about the shape of $\bar{X}$ and not *X*.

(b) The sample mean $\bar{x}$, sample standard deviation *s*, and standard error of the mean for the gas fill-up data are as follows:

Descriptive Statistics: Gas Fill-up (gal)

Variable	Total Count	Mean	SE Mean	StDev
Gas Fill-up (gal)	20	10.159	0.0207	0.0925

Construct and evaluate a two-sided 95% confidence interval for μ.

(c) Using the same data set to construct a 99% confidence interval for μ, the width of the interval:

A. Increases B. Decreases C. Is not affected D. Not enough information

Exercise 12

For sample size n = 64, use a standard normal table or Minitab to fill in the z-score necessary to construct a 97% confidence interval for μ.

$$\bar{x} \pm \_\_\_\_\_\_\_\_\_\_\_\_ * \frac{s}{\sqrt{64}}$$

Exercise 13

Bubble Gum Flavor

If students are helping to conduct this exercise in class, at least 30 students are needed to apply the concepts of this lesson. Instructions for creating the data set are below.* If you do not have time to collect class data or do not have at least 30 students to perform this activity, then you can use the 'Gum Flavor Time (sec)' data found in ***CIForMean_LargeSample_Activity.mtw***. Otherwise, you can put your students' data in column C7 of this worksheet.

* At the beginning of class, hand out a piece of bubblegum to each student. Tell them to record (in seconds) the length of time that they chew their gum before it loses its flavor. Ask them to go to the board as they are working on these exercises to record their flavor times. Once everyone has recorded his or her data, the instructor can enter the data in Minitab. The instructor can then email the worksheet data to students to ensure that everyone is working with the same data set.

(a) In Minitab, construct both a boxplot and histogram of the data. If you do not remember how to do this, see the ***Describing Data Graphically*** lesson.

(b) Describe the basic shape (symmetric, positively skewed, negatively skewed) and spread (amount of variability) of the data. It may not clearly fall into one specific category.

(c) Based on your answer to **(b)** and the sample size, can we assume $\bar{X}$ is normally distributed? Why or why not?

(d) In Minitab, construct a two-sided 90% confidence interval for μ.

(e) [BONUS] An investigator computes a 95% confidence interval for a population mean using a sample of size of n = 70. If she wishes to compute a 95% confidence interval that is half as wide, how large a sample does she need?