MATHEMATICAL ANALYSIS

PROBLEMS LIST 5

30.10.08

(1) Compute the partie=al sums $s_n = \sum_{k=1}^{n} a_k$, and then find $\lim_{n \to \infty} s_n$:

(a) $a_k = \dfrac{1}{a_k}$,   (b) $a_k = \dfrac{2^k + 5^k}{10^k}$.

(2) Prove that the series $\sum_{n=1}^{\infty} \dfrac{1}{2^n - 1}$ is convergent, and its sum is less than 2.

(3) Determine if the following series are convergent:

(a) $\sum_{n=1}^{\infty} \dfrac{1}{n^2 + 1}$,   (b) $\sum_{n=2}^{\infty} \dfrac{1}{n^2 - 1}$,   (c) $\sum_{n=1}^{\infty} \dfrac{1 + n}{n^2 + 1}$,

(d) $\sum_{n=1}^{\infty} \dfrac{2 \cdot 5 \cdot 8 \cdot \ldots \cdot (3n - 1)}{1 \cdot 5 \cdot 9 \cdot \ldots \cdot (4n - 3)}$,   (e) $\sum_{n=1}^{\infty} \dfrac{5n^2 - 1}{n^3 + 6n^2 + 8n + 47}$,

(f) $\sum_{n=1}^{\infty} \dfrac{1}{(2n - 1) \cdot 2^{2n-1}}$,   (g) $\sum_{n=1}^{\infty} \dfrac{1}{3n - 1}$

(h) $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n^2 + 2n}}$,   (i) $\sum_{n=1}^{\infty} \dfrac{1}{(n + 1)(n + 4)}$,

(j) $\sum_{n=1}^{\infty} \dfrac{1}{(2n + 1)!}$,   (k) $\sum_{n=1}^{\infty} \dfrac{n^2}{3^n}$,   (l) $\sum_{n=1}^{\infty} \dfrac{(2n - 1)!!}{3^n n!}$

($k!!$ denotes the product of all numbers not greater than k, of the same parity),

(m) $\sum_{n=1}^{\infty} \left(\dfrac{n}{2n + 1}\right)^n$,   (n) $\sum_{n=1}^{\infty} \dfrac{\left(\frac{n+1}{n}\right)^{n^3}}{3^n}$,   (o) $\sum_{n=1}^{\infty} \dfrac{1}{(n - 1)\sqrt{n + 1}}$,

(p) $\sum_{n=1}^{\infty} \sqrt{\dfrac{n + 1}{n}}$,   (q) $\sum_{n=1}^{\infty} \dfrac{n^2}{n!}$,   (r) $\sum_{n=1}^{\infty} \dfrac{n}{2n - 1}$,

(s) $\sum_{n=1}^{\infty} \dfrac{2^n}{n^4}$,   (t) $\sum_{n=1}^{\infty} \dfrac{1}{\sqrt{n^2 + n} - n}$,   (u) $\sum_{n=1}^{\infty} \dfrac{1000^n}{\sqrt[10]{n!}}$,

(v) $\sum_{n=1}^{\infty} \dfrac{\arctan n}{n^2 + \arctan n}$,   (w) $\sum_{n=1}^{\infty} \dfrac{3^n}{2^{2n}}$,   (x) $\sum_{n=1}^{\infty} \dfrac{n^3 + \pi}{n^\pi + e}$.

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(4) Which of the following series are convergent, and which are convergent absolutely:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}$,   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2 3^n}$,   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3}$,

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n+1}{n}$,   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{(n+4)(n+9)}}$,

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^{10n}}{3^{2n}}$,   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{n! \cdot (-5)^n}{n^n \cdot 2^n}$,

(h) $1-1+1-\frac{1}{2}-\frac{1}{2}+1-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}+\cdots+1-\overbrace{\frac{1}{k}-\frac{1}{k}-\cdots-\frac{1}{k}}^{k\ \text{times}}+\ldots$,

(i) $1-1+\frac{1}{2}-\frac{1}{4}-\frac{1}{4}+\frac{1}{3}-\frac{1}{9}-\frac{1}{9}-\frac{1}{9}+\cdots+\frac{1}{k}-\overbrace{\frac{1}{k^2}-\frac{1}{k^2}-\cdots-\frac{1}{k^2}}^{k\ \text{times}}$
$+\ldots$,

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n^3}{2^n}$,   (k) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n-\sqrt{n}}$,   (l) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^{n^2}}{n!}$,

(m) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin 77n}{n^2}$,   (n) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 17}{3^n}$,   (o) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n!+1}}{n!}$,

(p) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n^2}}{(n+3)^{1/4}}$,   (q) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2}{n(n+1)}(-1)^n$,

(r) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\left(1+\frac{(-1)^n}{\sqrt{n}}\right)$,   (s) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n\sqrt{4^n+3^n}}$,

(t) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+5\sqrt{n}+27}$,   (u) $\displaystyle\sum_{n=1}^{\infty} \frac{\binom{2n}{n}}{n!}$,   (v) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n^2}}{4^{\binom{n}{2}}}$,

(w) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/n}}$,   (x) $\displaystyle\sum_{n=1}^{\infty} \frac{\left(\frac{n+1}{n}\right)^{n^2}}{2^n}$,   (y) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\left(\frac{n+1}{n}\right)^{n^2}}{3^n}$,

(z) $\displaystyle\sum_{n=3}^{\infty} \frac{(\log n)^{\log n}(-1)^n}{n^{\log\log n}}$,   (ż) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\arctan n}$,

(ź) $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt{n+2}-\sqrt{n}\right)(-1)^n$.