Wrocław, 1 maja 2021

Exercise list no 8

Simulations and algorithmic applications of Markov chains

Exercise 1. Consider graph $G = (V, E)$ with $V = \{0, 1, 2, \ldots, N-1\}$ and $E = \{(i, i+1 mod N) : i \in \{0, 1, 2, \ldots, N-1\}\}$. Calculate the number of all valid configurations in a hard-core model on this graph.

Exercise 2. A chess queen can move (attack!) diagonally, horizontally or vertically. Consider a chess board of size 4x4. A configuration of queens is called valid if no two queens attack each other. Consider a Gibbs algorithm, which can move between configurations by making a legal chess move with one queen. What is the maximum number of queens such that this algorithm is irreducible?

Exercise 3. What is the minimal q such that Gibbs algorithm for q-colorings on full binary tree is irreducible?

Exercise 4. Let $G = (V, E)$ with $V = \{1, 2, \ldots, N\}$ and $E = \{(i, i+1 mod N) : i \in \{1, 2, \ldots, N\}\}$. Let π be a stationary distribution on V which is scaled geometric distribution (i.e. $\pi(i) = \frac{1}{C}p(1-p)^{i-1}, p \in (0, 1)$). Show Metropolis algorithm for this problem.

Exercise 5. Let G be a clique of size n. What is the minimal amount of edges to be removed such that the Gibbs algorithm for n-colorings of the graph G is irreducible.

Exercise 6. Let $E = \{0, 1\}^d$ be a state space. In each step we change one uniformly chosen coordinate with probability $\frac{1}{d+1}$ (from 0 to 1 or from 1 to 0). With the remaining $\frac{1}{d+1}$ we do nothing. Construct SST for this example.