Geometric and Asymptotic Group Theory II

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Blatt 1

(1) Check whether groups given by the following presentations are finite or not.
   (a) $\langle x, y, z \mid xzx^{-1}z^{-1}, yzy^{-1}z^{-1}, xyx^{-1}y^{-1}z^{-1}\rangle$;
   (b) $\langle s, t, u, w \mid s^2, t^2, u^2, w^2, [s, u], [s, w], [t, u], [t, w]\rangle$.

(2) Show that for a group G the following conditions are equivalent.
   (a) For every element $g \neq 1_G$ in G there exists a homomorphism $\varphi\colon G \to F$ into some finite group F, such that $\varphi(g) \neq 1_F$.
   (b) For every element $g \neq 1_G$ in G, there exists a finite index subgroup $K \leq G$ with $g \notin K$.
   (c) For every finite set A of nontrivial elements in G, there exists a homomorphism $\varphi\colon G \to F$ into some finite group F, such that $\varphi(g) \neq 1_F$, for every $g \in A$.
   (d) The intersection of all (normal) subgroups of G of finite index is trivial.
   (e) * Let $G = \pi_1(X, x_0)$. For every homotopically non-trivial loop γ in (X, x_0) there is a finite covering $p\colon \widetilde{X} \to X$ such that γ does not lift up to a loop in $\widetilde{X}$.

(3) Let T be a labeled tree of valence $k \geq 2$ (at every vertex). Let $G \leq \mathrm{Aut}(T)$ be the group generated by reflections wrt. edges. Show that G is residually finite.

(4) Show that the group $(\mathbb{Q}, +)$ is Hopfian, and $(\mathbb{R}, +)$ is not.