

Wstęp do topologii algebraicznej

Ćwiczenia 1

- (1) Show that the definition of the “cofinite topology” results in a topology.
- (2) Show that the weight of the *strzałka* topology is 2^{\aleph_0} .
- (3) Show that the interior and closure operations have the following properties:

$\overline{\emptyset} = \emptyset$	$\text{Int} X = X$
$A \subseteq \overline{A}$	$\text{Int} A \subseteq A$
$\overline{A \cup B} = \overline{A} \cup \overline{B}$	$\text{Int}(A \cap B) = \text{Int} A \cap \text{Int} B$
$\overline{(\overline{A})} = \overline{A}$	$\text{Int}(\text{Int} A) = \text{Int} A$
- (4) Show the following properties of the boundary operation: $\text{bd}(A \cup B) \subseteq \text{bd} A \cup \text{bd} B$, $\text{bd} A = \text{bd}(X \setminus A)$, $\text{bd}(\text{Int} A) \subseteq \text{bd} A$, $\text{bd} \overline{A} \subseteq \text{bd} A$. Argue that the inclusions may be strict.
- (5) Show that \mathbb{Q} is a dense boundary set in \mathbb{R} .
- (6) Give an example of a topological T_i space which is not T_{i+1} , for $i = 0, 1, 2$.
- (7) Show that a composition of continuous functions is continuous.
- (8) Show that a function is continuous iff it is continuous at every point.