Abstract

The thesis consists of two independent chapters.

Chapter 1. Let M be an n-dimensional manifold with a given Riemannian metric. By $\widetilde{M}$ we denote the universal cover of M endowed with the pullback metric. We say that M is macroscopically large if one cannot map $\widetilde{M}$ into $(n-1)$-dimensional simplicial complex such that preimages of points have uniformly bounded diameters.

The notion of macroscopic dimension was introduced by Gromov in order to study manifolds with positive scalar curvature.

Let $f\colon M \to B\pi_1(M)$ be a map classifying the universal cover. Suppose that M is oriented. Denote by $[M]$ the fundamental class of M. We say that M is rationally inessential if $f_*([M]) = 0 \in H_n(B\pi_1(M); \mathbf{Q})$.

In [9] A. Dranishnikov conjectured that rationally inessential manifolds are not macroscopically large. In this thesis we present counterexamples to the Dranishnikov conjecture. In order to construct such manifolds we use Davis complexes of right-angled Coxeter groups and the theory of small covers. The Dranishnikov conjecture, in trivial way, would imply Gromov's weak positive scalar curvature conjecture for rationally inessential manifolds. The existence of the manifolds we construct here shows that Gromov's weak conjecture may be nontrivial even for rationally inessential manifolds. Here we establish Gromov's conjecture (the strong version) for a subclass of manifolds we construct. In general the conjecture is open.

Chater 2. In this chapter we are interested in homological construction of aperiodic systems of tiles for certain Riemannian manifolds. We remind a classical procedure of Block-Weinberger which provides such a system when manifold admits an isometric, cocompact action of an non-amenable group. We show that sometimes when the acting group is amenable (e.g., for manifolds admiting isometric, cocompact action of Grigorchuk groups) a modification of Block-Weinberger technique leads to new systems of aperiodic tiles.