

Abstract

This dissertation presents a comprehensive mathematical and numerical analysis of non-linear population dynamics models incorporating non-local dispersal in bounded spatial domains. Motivated by the complex dynamics of semi-arid ecosystems, such as vegetation pattern formation and abrupt desertification, the study replaces classical local diffusion (the Laplace operator) with non-local integral operators. The research considers both scalar reaction-dispersal equations and hybrid reaction-diffusion-dispersal systems, exemplified by a non-local formulations of the Logistic model with Allee effect and Klausmeier-Gray-Scott model, under closed (Neumann-type) and hostile (Dirichlet-type) boundary conditions.

The author's contributions are categorized into three main areas: theoretical analysis, numerical methods, and theoretical ecology.

In the realm of functional analysis, the dissertation establishes a rigorous spectral theory for non-local dispersal operators. It provides an explicit characterization of the continuous spectrum, proves the existence of a strictly positive spectral gap under specific geometric and kernel assumptions, and formulates an exact max-min characterization of the discrete spectrum and the principal eigenvalue for both boundary conditions. Building upon this spectral foundation, the study investigates time-independent stationary problems, establishing criteria for the existence and uniqueness of spatially heterogeneous stationary states. It rigorously proves the emergence of discontinuous spatial patterns under highly non-local dispersal regimes and utilizes bifurcation theory to construct continuous patterned solutions. Furthermore, the well-posedness, global-in-time existence, and continuous dependence on initial data for the associated evolutionary Cauchy problems are established. By constructing invariant regions, the author provides sharp analytical conditions for the long-time asymptotic behavior of populations, distinguishing between exponential extinction and uniform persistence.

To overcome the computational intractability of dense convolution matrices inherently arising from non-local operators, an efficient pseudo-spectral numerical scheme is developed. This method exploits orthogonal polynomial expansions (such as Legendre polynomials) to drastically reduce computational complexity, enabling high-resolution 1D and 2D simulations. The dissertation includes a rigorous convergence analysis, establishing exact error bounds for the approximation of the continuous convolution operator, the local multiplicative functions, and the temporal integration errors.

Finally, the mathematical and numerical frameworks are applied to pressing ecological questions. The study formally translates biological principles into mathematics by defining the Critical Patch Size and Critical Aggregation Threshold for non-local models. Extensive numerical continuation and bifurcation analysis demonstrate that ecosystems governed by long-range, heavy-tailed dispersal kernels exhibit significantly enhanced resilience, per-

sisting in smaller and more fragmented habitats compared to those restricted by local diffusion. The research also uncovers unique spatial morphologies induced by non-local interactions, notably the “boundary drop-off” and “boundary ridge” phenomena, which are entirely absent in classical local reaction-diffusion models, providing profound insights into the structural self-organization of species in fragmented habitats.