# On some properties of solutions to the stochastic recurrence equation

Abstract of the PhD thesis


The subject of the thesis is regularity of solutions to the stochastic recurrence equation

$$\mathbf{X}_{n+1} = \mathbf{A}_{n+1}\mathbf{X}_n + \mathbf{B}_{n+1}, \tag{0.2}$$

where $(\mathbf{A}_n, \mathbf{B}_n)$ is an i.i.d. sequence of random vectors. Here regularity is understood in two ways. The first four chapters of the thesis concern the tail assymptotics

$$\mathbb{P}(X_i > t)$$

in multidimensional case, where $X_i$ denotes the $i$-th component of the stationary solution $\mathbf{X}$. Chapter 5 describes a method of reduction of study of solutions to (0.2) with $d$-dimensional matrices $\mathbf{A}_n$ to the case when $\mathbf{A}_n$ are block triangular matrices. Chapter 6 concerns absolute continuity of the stationary solution in the univariate case.

In chapters 1-4 we assume that $\mathbf{A}_n$ are $d$-dimensional upper triangular matrices with nonnegative entries and that there are positive $\alpha_1, \dots, \alpha_d$ such that

$$\mathbb{E}A_{ii,n}^{\alpha_d} = 1.$$

Under assumption $\alpha_i \neq \alpha_j$ for $i \neq j$ we find (Chapter 3) the exact assymptotics of tails of stationary solution, i.e. positive numbers $C_i, \widetilde{\alpha}_i$ such that

$$\lim_{t\to\infty} t^{\widetilde{\alpha}_i}\mathbb{P}(X_i > t) = C_i.$$

Next we weaken the assumption on the diagonal entries of $\mathbf{A}_n$: we allow $\alpha_i = \alpha_j$ for $i \neq j$, but exclude $A_{ii,n} = A_{jj,n}$ a.s. Under such assumptions we show (Chapter 4) the lower and upper bounds for the tails of stationary solution, i.e. we show that there are positive constants $M_i, L_i, T$ such that

$$M_i t^{-\widetilde{\alpha}_i} \le \mathbb{P}(X_i > t) \le L_i t^{-\widetilde{\alpha}_i}(\log t)^{\xi(i)}$$

for $t > T$, where $\xi(i)$ is a parameter depending on the law of $\mathbf{A}_n$.

In Chapter 6 we assume that $\mathbf{A}_n$ are real random variables and we find nontrivial condidons yielding absolute continuity of solutions of the solution $\mathbf{X}$ with respect to the Lebesgue measure.


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