

Abstract

The dissertation addresses problems in multivariate statistics related to the estimation of covariance matrices, where observed random vectors can naturally be divided into groups, and the dependence structure both within and between these groups is reflected in the block structure of the covariance matrix. Introducing structural assumptions substantially reduces the number of unknown parameters and leads to more stable and computationally efficient estimators, particularly in high-dimensional settings where classical estimation methods often fail.

The dissertation focuses on block-structured covariance matrices partitioned into four sub-blocks, with square, symmetric diagonal blocks and rectangular off-diagonal blocks. The first part of the dissertation concerns the identification of covariance structures among candidate block-structured covariance matrices using the entropy loss function as a discrepancy criterion. A block-diagonal structure with compound symmetry and a first-order autoregression structure in the off-diagonal blocks are considered.

The main objective of the dissertation is to develop positive definite and well-conditioned estimators preserving the assumed block structure, while remaining computationally feasible and possessing desirable statistical properties. The estimation of the aforementioned covariance structures is carried out using classical maximum likelihood as well as a least squares technique. To ensure the positive definiteness of the least squares estimator, its regularization is introduced by adapting a shrinkage method to matrices with a prescribed structure. The proposed methodology is subsequently extended to covariance structures with structured diagonal blocks. Within this framework, it is shown that the optimal choice of the target matrix in the shrinkage method is the projection onto a quadratic subspace of the space of covariance matrix structures. Consequently, particular attention is devoted to quadratic spaces of partitioned matrices, and a general characterization of quadratic spaces is established. The projection onto an arbitrary quadratic subspace is derived, and methods for constructing nonzero off-diagonal block structures that preserve quadratic structure are presented. All the estimators introduced in the dissertation are compared with respect to their statistical properties, conditioning, and computational complexity, with the use of simulation studies conducted in the R environment.

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