Abstract

This dissertation primarily concerns Banach spaces of real-valued continuous functions on a compact space. We usually consider compact spaces that are linearly ordered, the so-called compact lines. Most of the questions are formulated in the language of functional analysis; however, their solutions usually require methods from topology, set theory, and measure theory. In this work, we analyse both classical problems that are still actively studied, as well as problems that have been posed relatively recently.

We begin with Chapter 3, in which we investigate the properties of extension operators $E : C(K) \to C(L)$ for certain pairs of compact spaces $K \subseteq L$. This topic is related to the problem of the existence of certain short exact sequences, which fits into the broader trend of applying homological methods in the Banach space theory. During the chapter, we introduce a combinatorial object similar to gaps and prove some of its properties, thus revealing certain structural aspects of sequences of measures on compact lines.

In Chapter 4, we define a new dimension for Banach spaces which, in a specific case, distinguishes between spaces of continuous functions on products of non-metrizable compact lines with different numbers of factors. This stands in contrast to the metrizable case, where Milutin's classical theorem states that any two spaces of continuous functions on uncountable compact metrizable spaces are isomorphic.

We conclude with Chapter 5, where we present estimates for the Banach–Mazur distance between certain classical spaces of continuous functions. On the one hand, we address questions posed by Bessaga and Pełczyński [**12**] concerning the distance when the compact spaces under consideration are countable. On the other hand, we study the distance between the classical spaces ℓ_∞ and $L_\infty[0,1]$, which opens several interesting directions for further research.

Keywords: space of continuous functions, compact line, extension operator, almost chain, martin's axiom, Banach spaces not isomorphic to their squares, Banach-Mazur distance, injective space