Abstract

Branching processes are a rapidly developing area of probability theory. Initially introduced to describe simple birth-and-death dynamics, they have subsequently found numerous applications in physics and biology, including models of nuclear reactions and epidemic spread.

The classical branching random walk starts with a single particle located at the origin. At time 1, this particle dies and is replaced by a random number of randomly placed offspring. At the next time step, every particle repeats the process and is again replaced by a random number of descendants whose positions are determined by a random displacement of the parent's position. The process is then iterated indefinitely. In this thesis, we investigate two modifications of branching random walks: multi-type models and perturbed models.

The multi-type branching random walk is a natural generalization of the standard model, allowing for particles belonging to distinct classes. These classes determine both the offspring distribution and the displacement law of each particle. This framework enables the description of more complex phenomena, such as the dynamics of cell populations with different phenotypes. The interaction between different types may lead to surprising results, including propagation at significantly higher speeds than in any of the corresponding single-type models.

The perturbed branching random walk introduces an additional source of randomness in determining particle positions, providing greater flexibility in modeling stochastic systems. The long-term behavior of such models can differ significantly from that of the classical setting.

The asymptotic behavior of the maximal position has been a central topic of research in the context of classical branching random walks, as it gives fundamental insights into the long-term dynamics of the process and allows for a deeper understanding of complex mechanisms arising in biology, physics, and epidemiology. The principal objective of this thesis is to provide as general as possible a description of this behavior in both models mentioned above.