Abstract

Research on the distributional properties of extremes of stochastic processes has attracted growing attention in recent literature. These studies are motivated by both the theoretical aspects of extreme value theory and applied probability problems, including finance, physics, risk theory, and queueing theory. In recent years, the asymptotic behavior of various functionals of stochastic processes was intensively studied for both Gaussian and Lévy models, which commonly describe a broad class of phenomena and require different proof techniques based on the specific distributional properties of each of class of stochastic processes.

Early studies mainly focused on one-dimensional problems for supremum, infimum and sojourn-time functionals. Contemporary works, such as, emphasize developing methods for general functionals of stochastic processes, often multidimensional, and on extremes over random time horizons.

In the most general form, the problems analyzed in the aforementioned literature can be formulated as investigation of

$$\mathbb{P}\{\Theta(\{X(\boldsymbol{t}) : \boldsymbol{t} \in E\}) \in K_u\},$$

where $\{X(\boldsymbol{t}) : \boldsymbol{t} \in \mathbb{R}^d\}$ is a stochastic process or a field, $E \subset \mathbb{R}^d$, $\Theta(\cdot)$ is a $\mathbb{R}^q$-valued functional, and where $K_u \subset \mathbb{R}^q$ is a family of sets depending on u (usually $K_u = [u, \infty)$) satifying the *rare event regime*, i.e. where the above probability goes to zero as u tends to infinity.

The analysis of the extremes of each class of the aforementioned processes needs distinct techniques. The research of Gaussian processes is based on the Pickands lemma and the double-sum method, and the Borell-TIS inequality, as well as comparison inequalities (e.g., the Slepian, Gordon, and Sudakov-Fernique inequalities), whereas the study of Lévy processes relies on Doob's techniques, stopping-time arguments, the Markov property, the exponential change of measure technique, and the Wiener-Hopf factorization, among others.

This dissertation investigates specific problems related to risk and queueing models. It introduces new techniques, such as an extension of Breiman's lemma, and develops known methods to prove results in various areas of applied probability, as described below. In the presented dissertation:

1. We calculated the asymptotic behavior of the cumulative Parisian ruin probability for correlated Brownian motion with drift under the many-source regime.

2. We determined the asymptotics of the buffer overflow probability for the stationary Gaussian queues with fractional Brownian motion as input over a random time interval $[0, \mathcal{T}_u]$, where $\mathcal{T}_u$ is a non-negative random variable that may depend on u.

3. We derived the tail asymptotics for a broad class of functionals of stationary Lévy queues with light-tailed input; the proof is based on the celebrated Breiman lemma.

4. We extended Fougeres & Mercadier's version of Breiman's lemma on products of the random matrices and the random vectors indexed by u (and possibly by an additional parameter τ_u), and applied this result to derive exact asymptotics for several Gaussian models. In particular, we provided independent proofs of the celebrated Pickands and Piterbarg lemmas, as well as the uniform Pickands lemma for homogeneous functionals of Gaussian fields. Our results generalized the previous ones to a broad class of threshold sets of the form $u + \frac{1}{u}\log(K)$, whereas the previous results only covered the special case $K = [1, \infty)$. Additionally, we calculated the asymptotics of the supremum of self-standardized Gaussian processes, which are related to Gamma bridge and Gaussian processes with random variance.