Abstract

This doctoral dissertation concerns combinatorial Banach spaces, that is, Banach spaces induced in a certain way by families $\mathcal{F}$ of finite subsets of $\mathbb{N}$ (or other infinite countable set). These spaces are denoted by $X_{\mathcal{F}}$. The thesis consists of four parts.

In the first part, we introduce the necessary notions, theorems, and facts that we use in the following chapters.

In the second part, we introduce various examples of combinatorial spaces. We investigate how combinatorial properties of families influence the structure of the spaces they induce. Particular attention is devoted to spaces associated with non-compact families, a subject for which the existing literature is rather sparse. In particular, we construct an example of an ℓ_1-saturated space failing the Schur property, and we provide a description of Pełczyński's universal space as a combinatorial space.

In the third part, we study the dual spaces of the combinatorial Banach spaces generated by compact families $\mathcal{F}$. Our aim is to obtain a convenient, equivalent description of the norm on the dual space. To do this, we introduce a quasi-Banach space $X^{\mathcal{F}}$ which, as it turns out, shares many properties with $X_{\mathcal{F}}^*$. In particular, we show that this quasi-Banach space provides yet another example of an ℓ_1-saturated space without the Schur property. Moreover, we prove that the Banach envelope of $X^{\mathcal{F}}$ is isometrically isomorphic to $X_{\mathcal{F}}^*$.

In the fourth part, we investigate the extreme points of the unit ball in combinatorial spaces and in related spaces. We provide characterizations of extreme points in several concrete cases. In addition, we adress the problem of describing extreme points in spaces induced by graphs.