Abstract of doctoral dissertation

STOCHASTIC SADDLE POINT METHOD IN RESERVES MODELING

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In the doctoral dissertation "Stochastic saddle point method in reserves modeling" we examine asymptotics for some compound distributions. In the paper we consider the random sum

$$A = \sum_{j=1}^{N} U_j,$$

where N is a Poisson random variable with mean equal to a and $U_1, U_2, \ldots$ are random variables, independent from N, identically distributed with distribution B. Moreover we assume that $U_1, U_2, \ldots$ are nonnegative and have integer values. We will look for the asymptotic for expressions $\mathbb{P}(A = k)$ and $\mathbb{E}[\eta(N)|A = k]$, when $k \to \infty$, for some function η. The stochastic saddle point method will be used. For that reason we introduce a family of probabilistic measures $\mathbb{P}^s$. The new probabilistic measure is choosen in such way, that for every function $\eta \geq 0$ the following equality holds

$$\mathbb{E}[\eta(N)|A = k] = \mathbb{E}^s[\eta(N)|A = k].$$

We present the asymptotics, which are expressed using introduced probability measure $\mathbb{P}^{(\theta)}$, where θ i the saddle point defined as a solution of the following saddle point equation

$$\mathbb{E}^{\theta} A = k. \tag{1}$$

We consider in details examples where random variables U_j have a distribution with bounded support and where we deal with logconcave class of distributions. The following hypothesis is presented

$$\mathbb{E}[N|A = k] \sim \mathbb{E}^{\theta} N, \quad \text{for } k \to \infty,$$

where θ is as saddle point, which solve the equation in (1). We look into examples for specific distributions.

In the second part of the doctoral dissertation we present the conception of a stochastic model for valuation of reserves, which was a motivation to examine the asymptotics for the random sums in the first part. In proposed model N is non-homogeneous Poisson process describing claims occurences. At each claim occurence time T_i, a process X_i is started, which will be interpreted as claim payment stream:

$$X_i(t) = \sum_{j=1}^{M_i(t)} C_{ij},$$

where M_i is another non-homogeneous Poisson process describing the following moments of payments for claim occured at time T_i, and C_{ij} are random variables which refere to

the value of the following payments for that claims. Cumulated value of payments up to time t for the claims which occured in the first year will be caluclated by the following formula:

$$S(t) = \sum_{i=1}^{N(1)} X_i(t - T_i).$$

We propose predictors for reserve value in the form of conditional expectation or predictiors based on the quantiles in the appropriate conditional distribution. The most important point in this part of the paper was the analysis of the following conditional expectation:

$$\mathbb{E}\left[N(1)\middle|\sum_{j=1}^{N(1)} W_j = k\right], \tag{2}$$

where the random variables W_i have a mixed Poisson distribtion with a mixing parameter $b(t-T_i)$, where T_i as definded. The methods presented in the first part of the paper were used to deliver the approximation for the expected value in (2).

In the last part we present numeric methods, which may be used for performing calculation for the formulas arising in stochastic model considered in the third chapter of the dissertation. The Fast Fourier Transform method is proposed. It is presented how to use it in the examined case. The calculation for specific examples were performed, and the results were confronted with the values based on the proposed hypothesis for the asymptotics.